On Minimal Trees in Polygonal Maps

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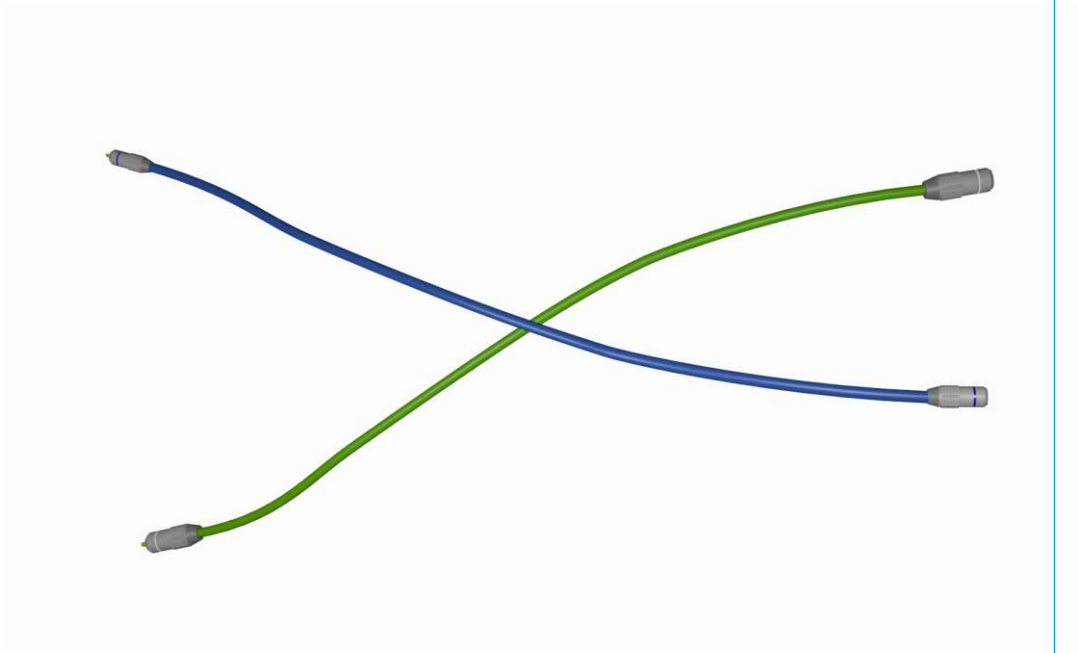
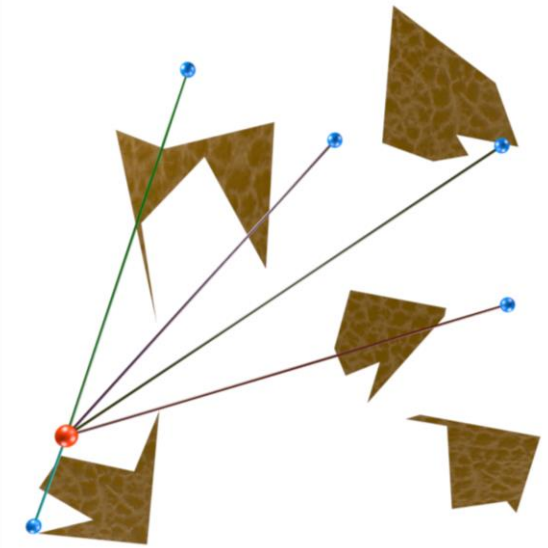
Waseda University

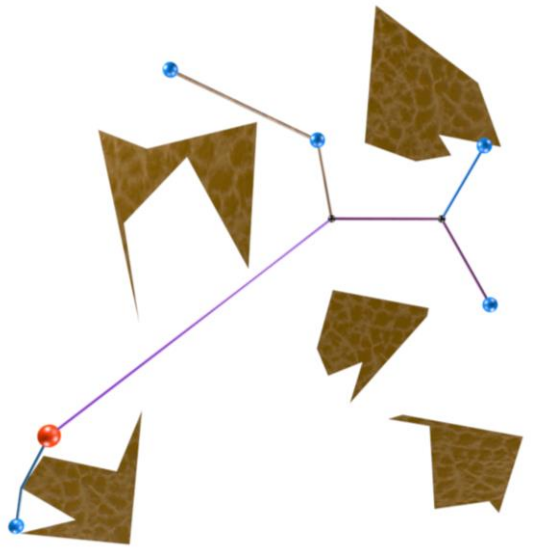
15th Annual Humies Awards

2018

Minimal Trees
What is it?





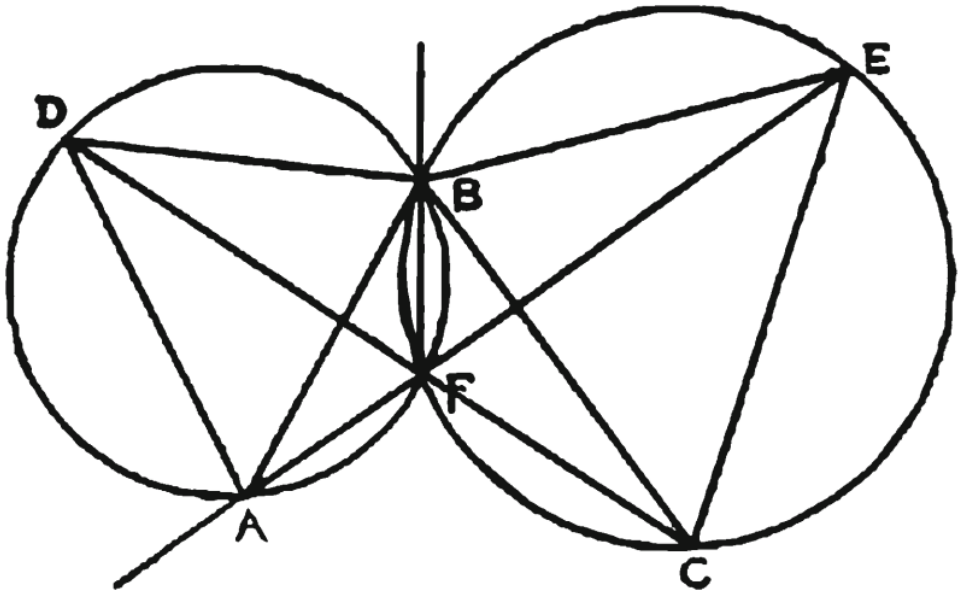


What is the network of minimum length interconnecting *n* points in the Euclidean plane?

datis tribus punctis, quartum reperire, a quo si ducantur tres rectae ad data puncta, summa trium harum rectarum sit minima quantitatis de Fermat ~1643

given three given points, a fourth is to be found, from which if three straight lines are drawn to the given points, the sum of the three lengths is minimum

Pierre de Fermat
(1661 - 1665)
Source: Wikipedia



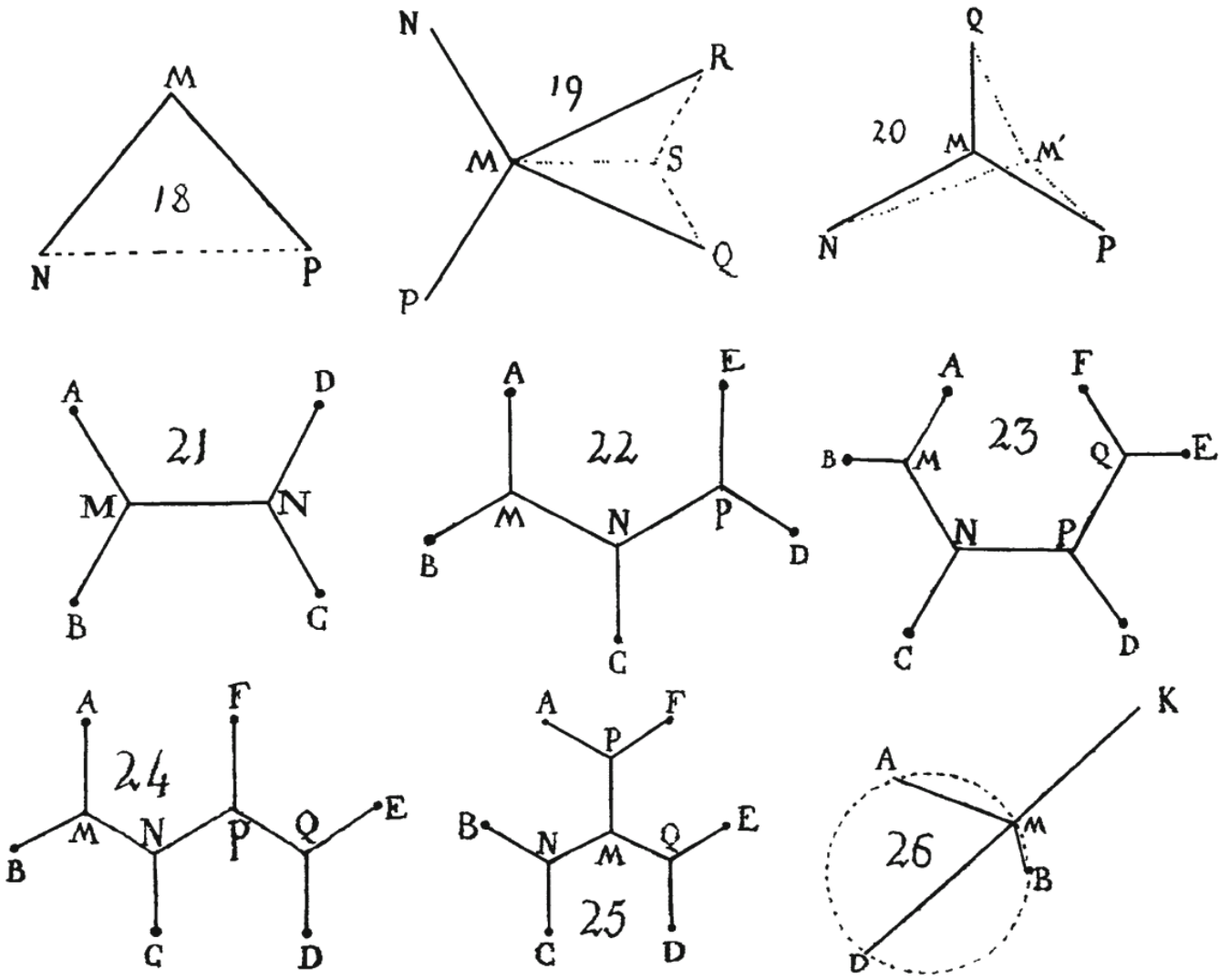
[Fig. 38].

Viviani ~1659

Evangelista Torricelli
(1608 - 1649)
Source: Wikipedia

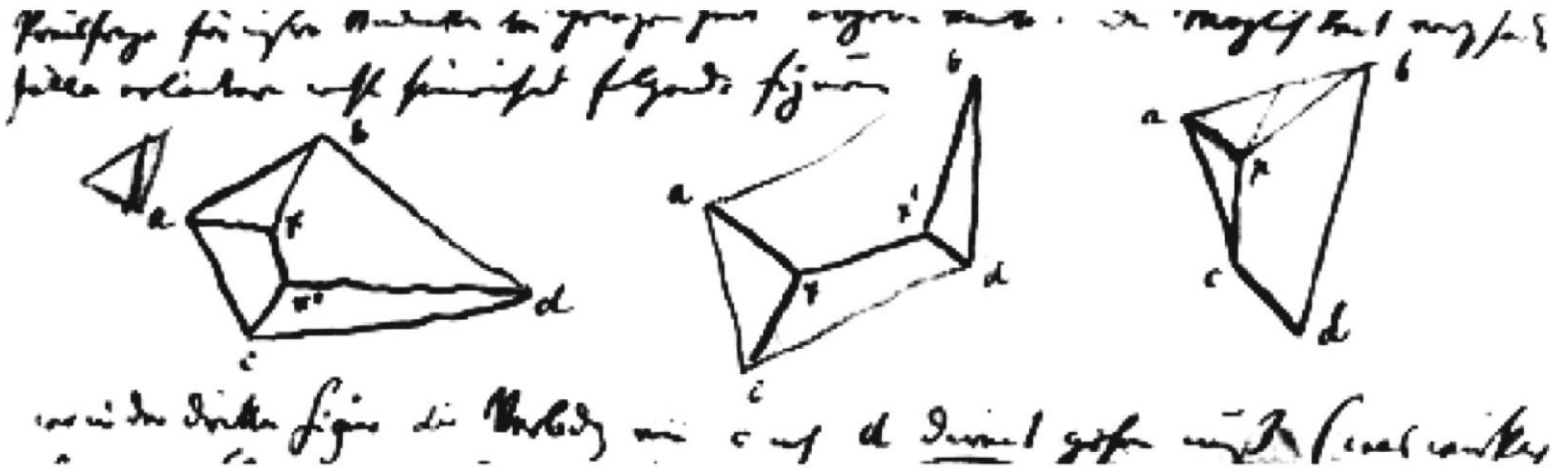
Joseph Diaz Gergonne (1771-1859)

Source: Wikipedia



Gergonne, 1810

Carl Friedrich Gauss
(1777 - 1855)
Source: Wikipedia

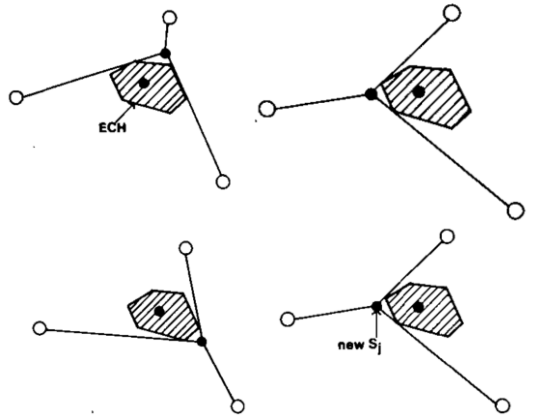


Gauss Letter to Schumacher, 1836

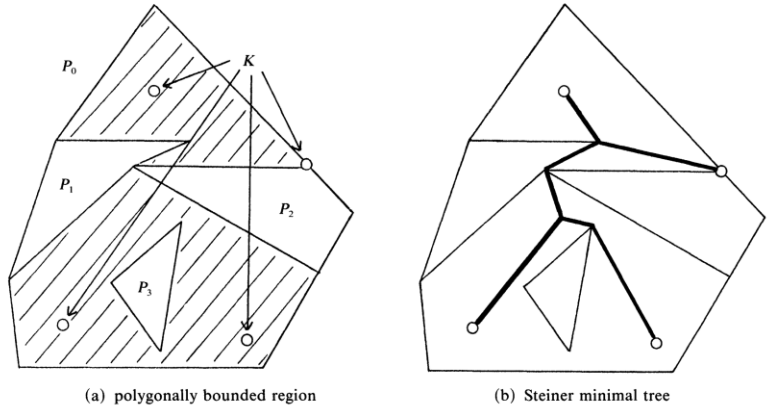
Source: @thatsmaths

Minimal Trees with Obstacles

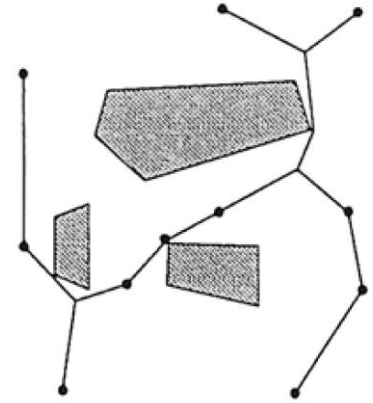
Brazil and Zachariasen, 2015



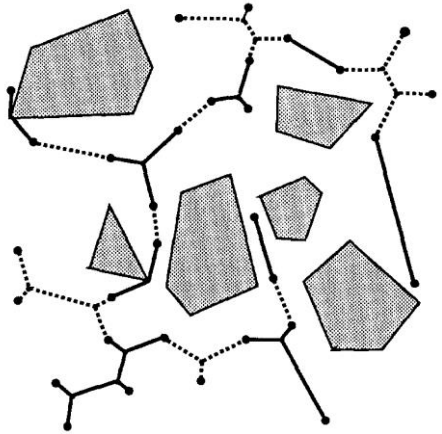
Macgregor and Liebman, 1979



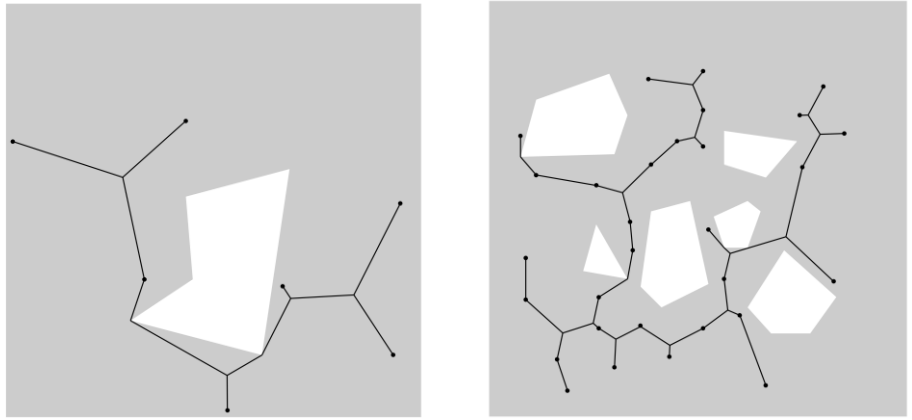
Provan, SIAM, J. Compu. 1988



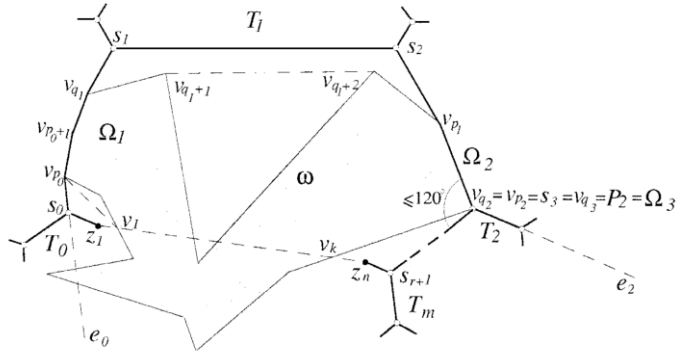
Winter and Smith, 1991



Winter, 1993



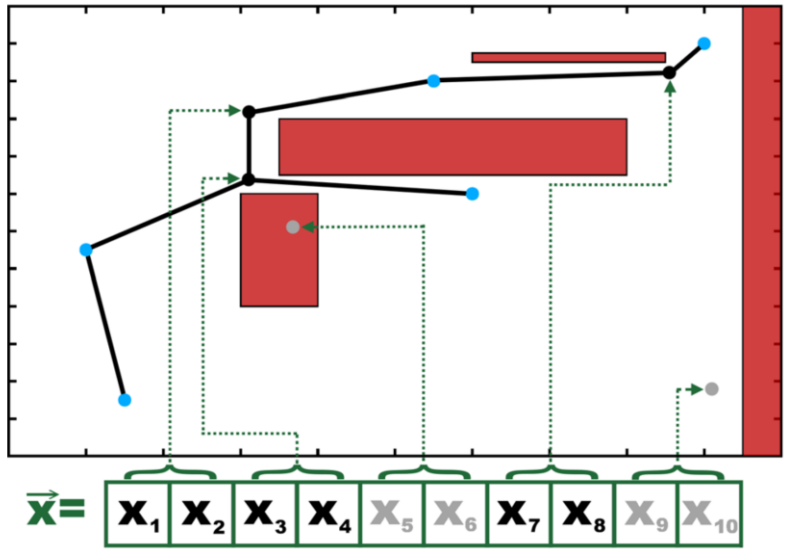
Zachariasen and Winter, ALENEX, 1999

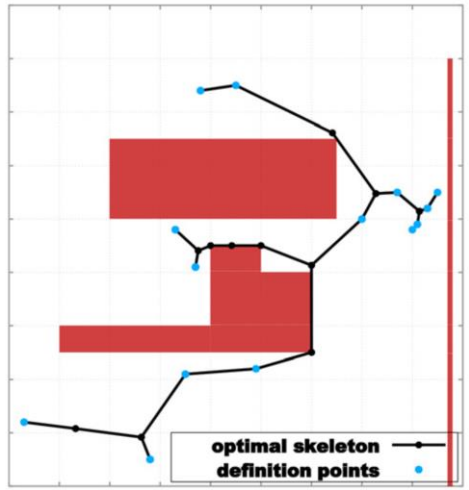


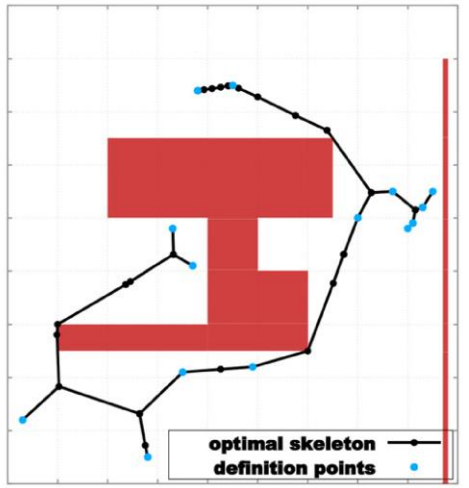
Weng and Smith, 2001

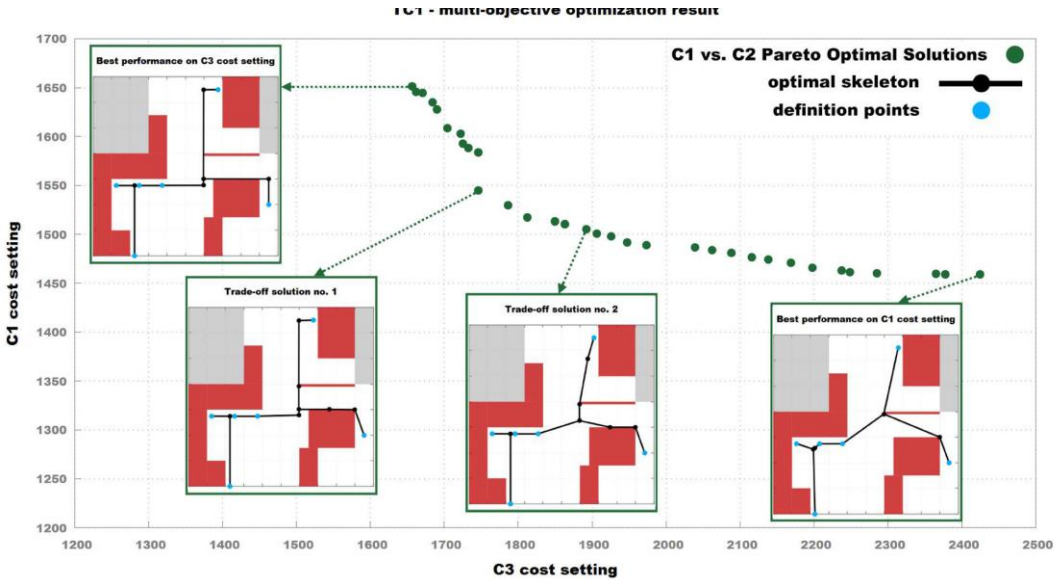
Minimal Trees with Obstacles

Alexandru-Ciprian, et al, 2018



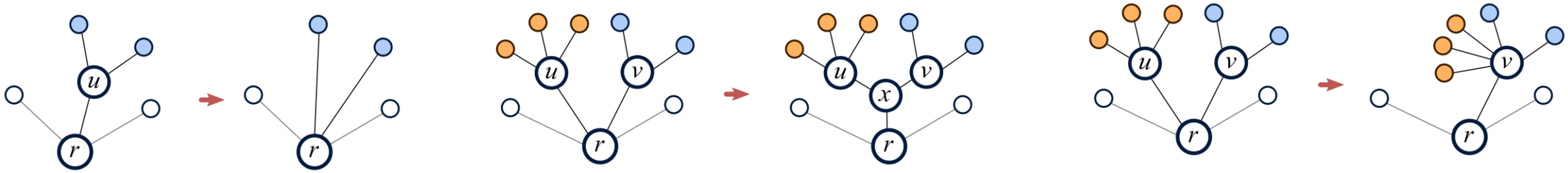




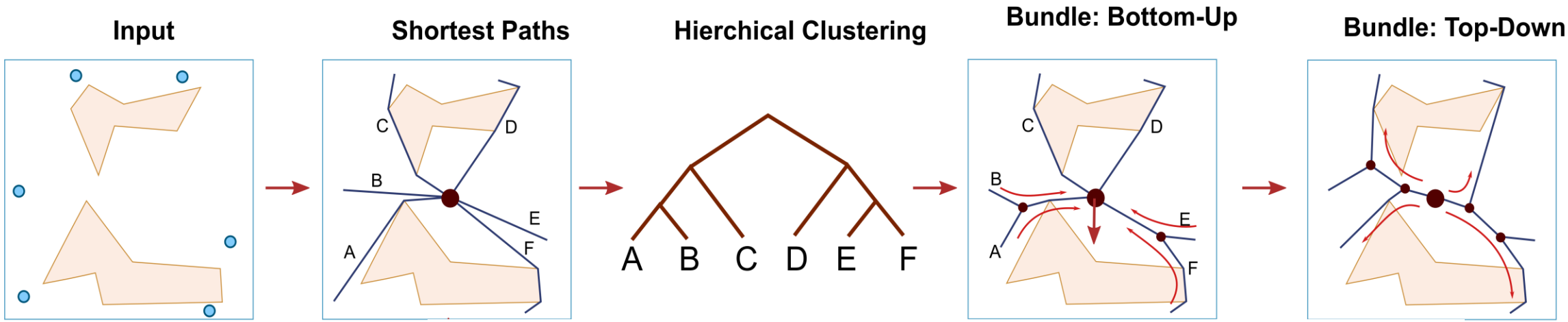


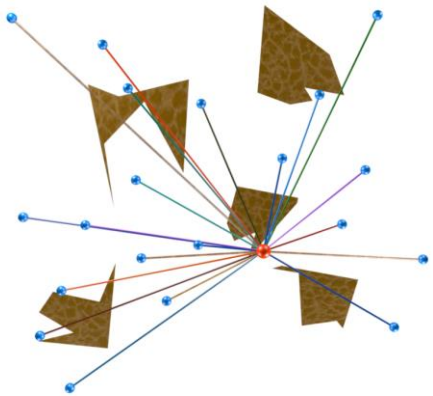
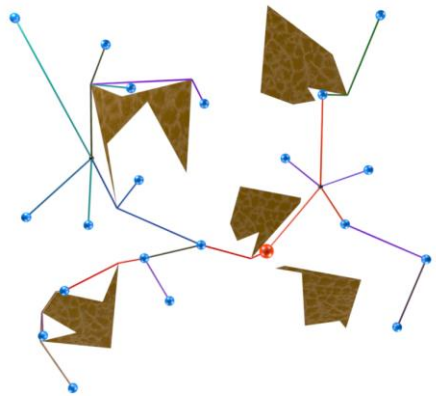
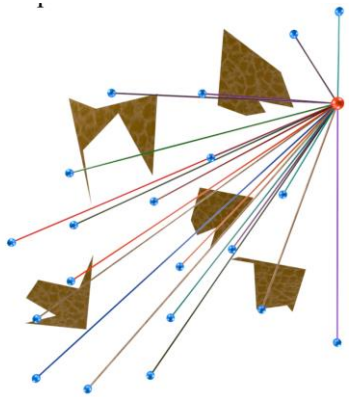
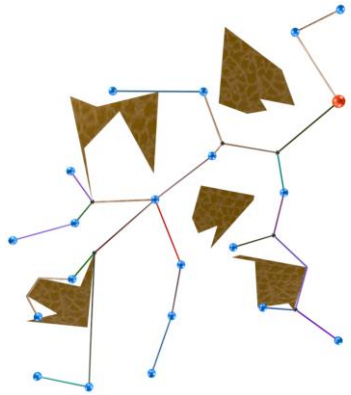
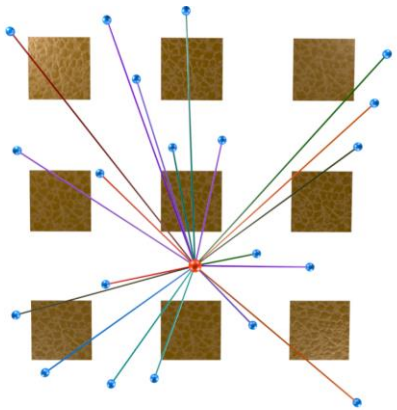
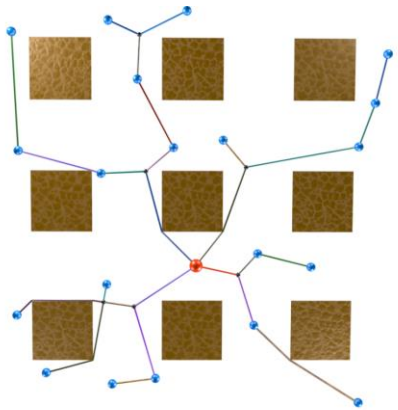
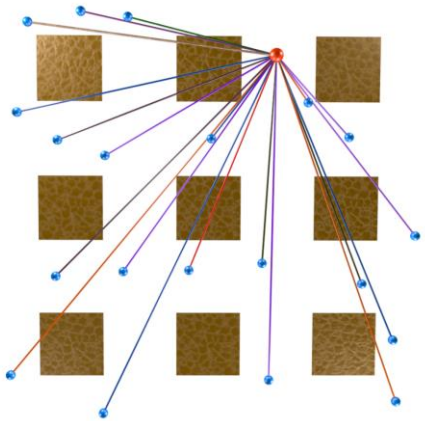
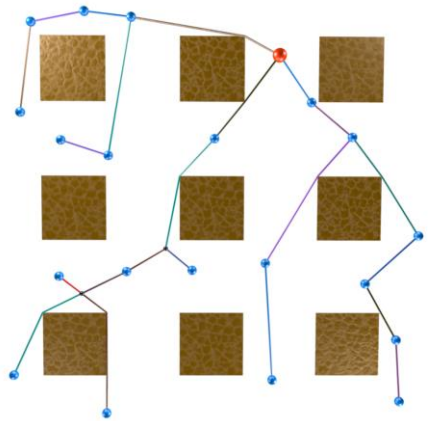
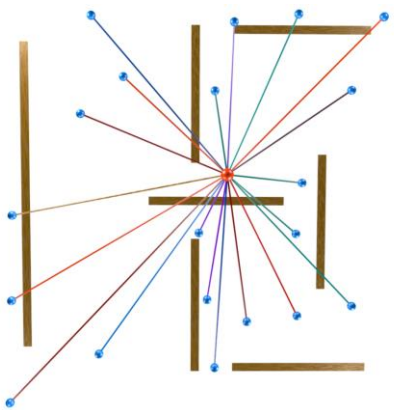
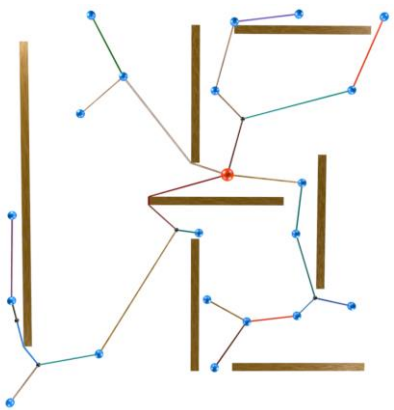
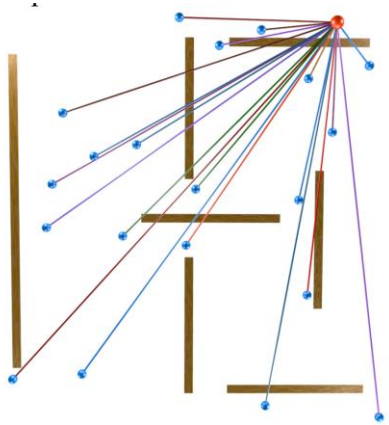
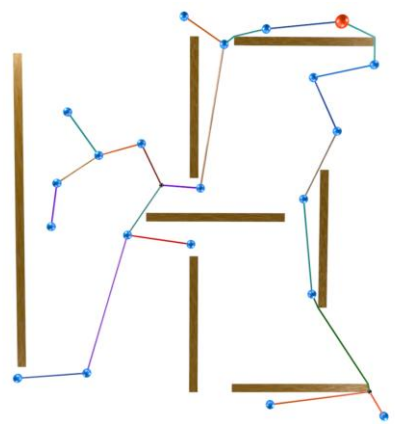
Can Evolution render Minimal Trees?

On nature inspired computing



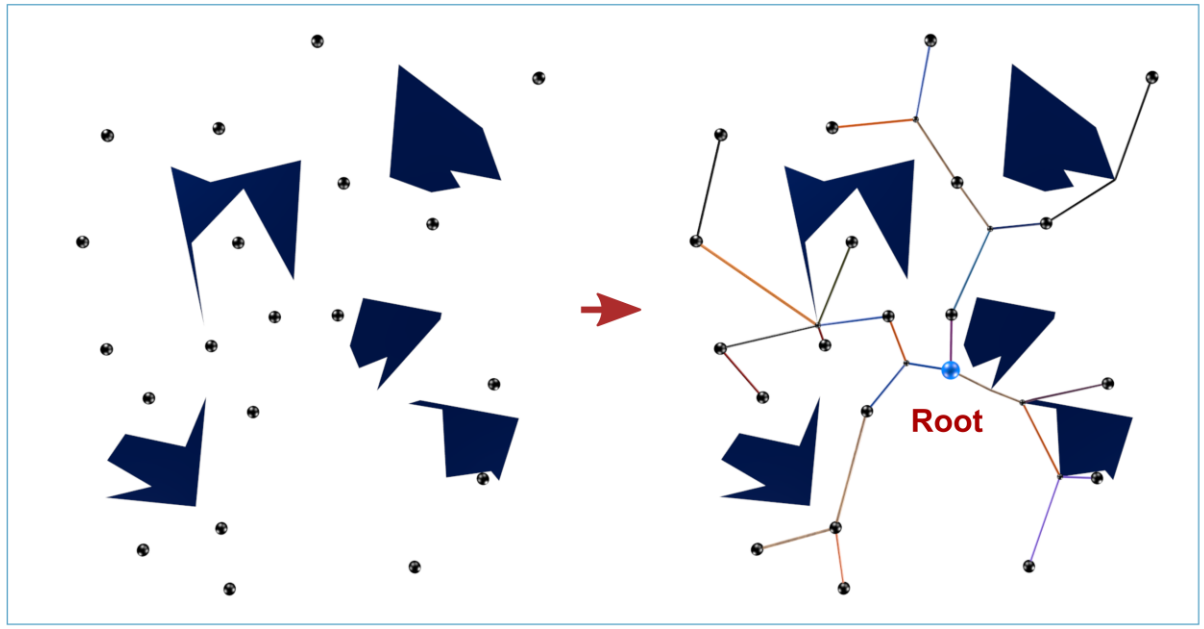
Expand **Shrink** **Merge**



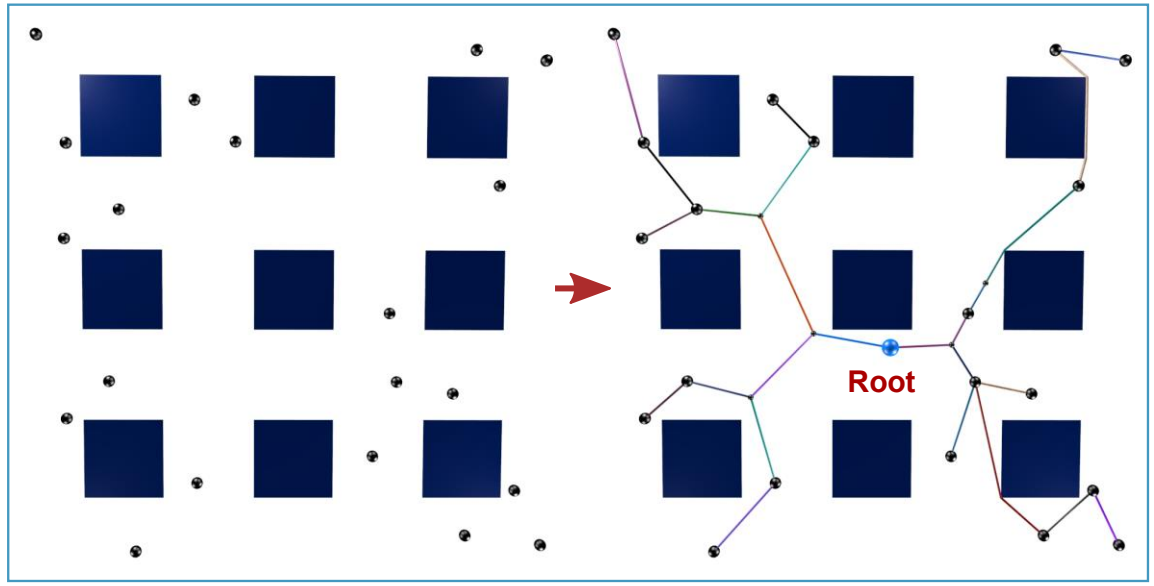
Input

Minimal Tree



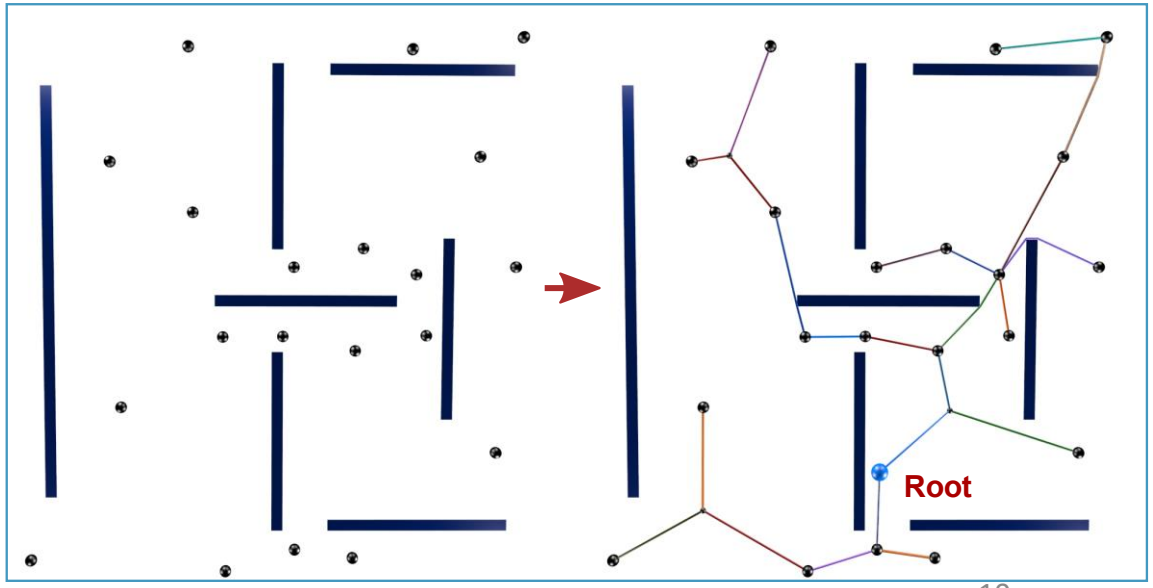
Input

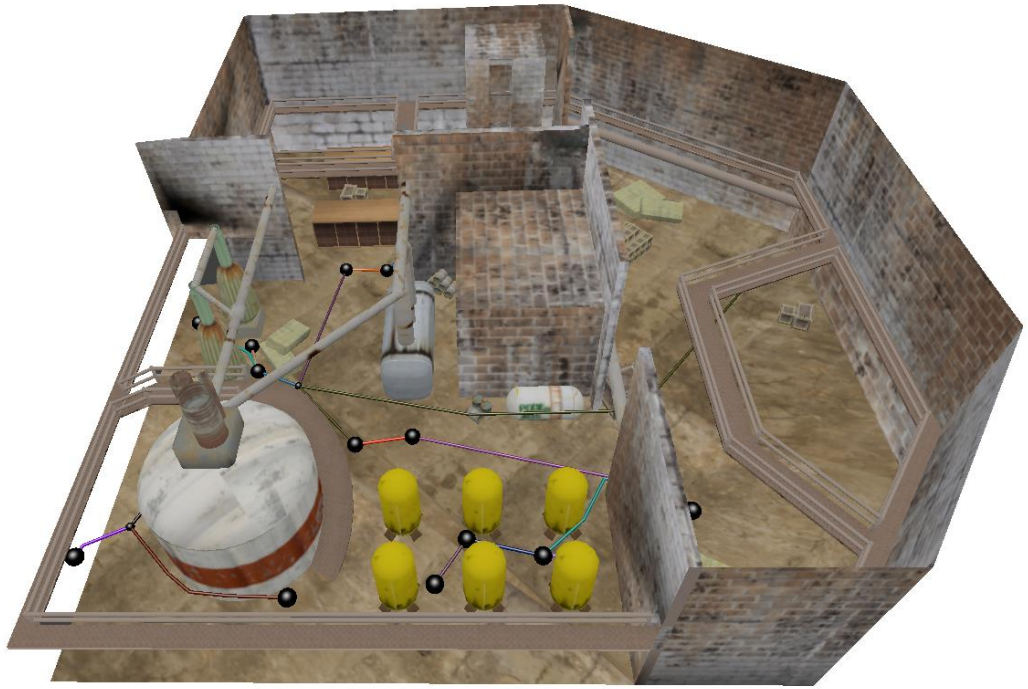
Minimal Tree

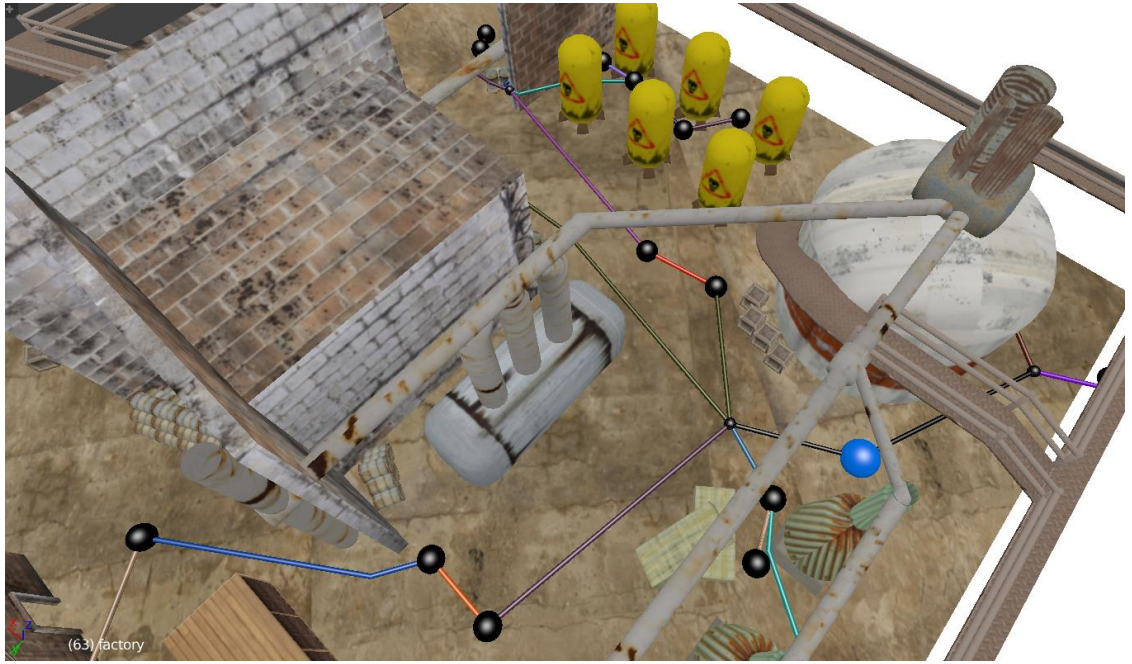


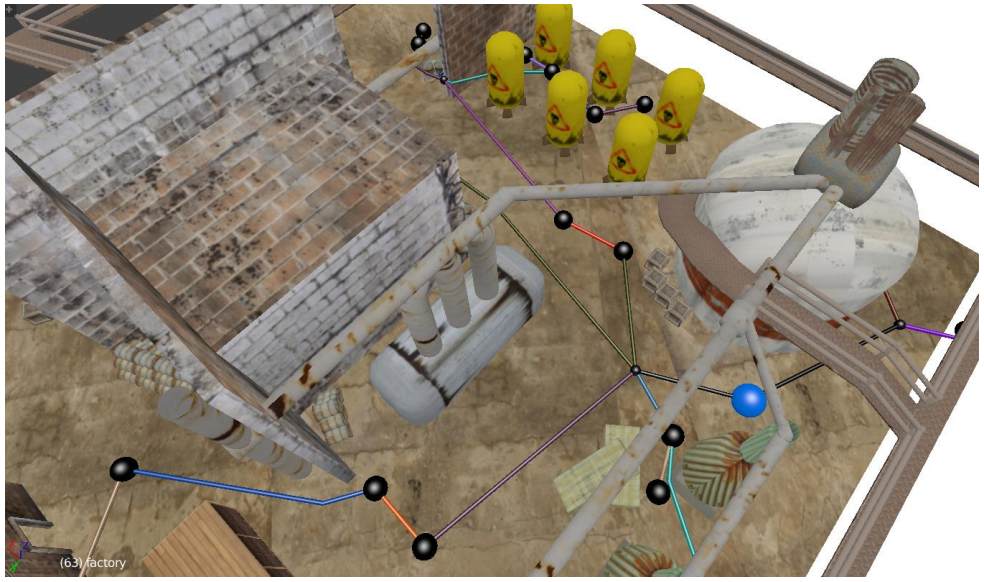
Input

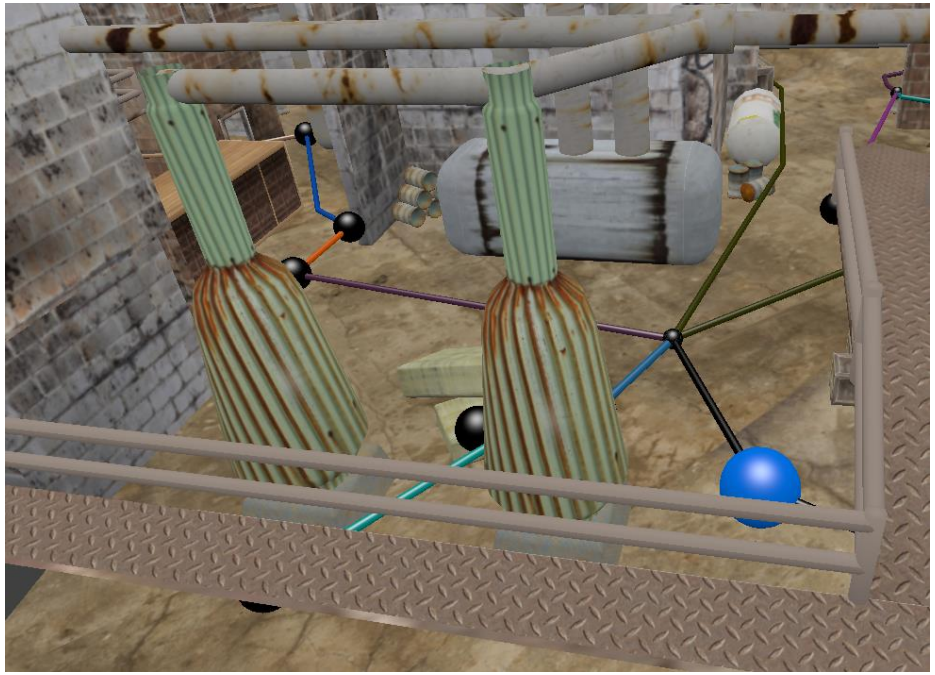
Minimal Tree

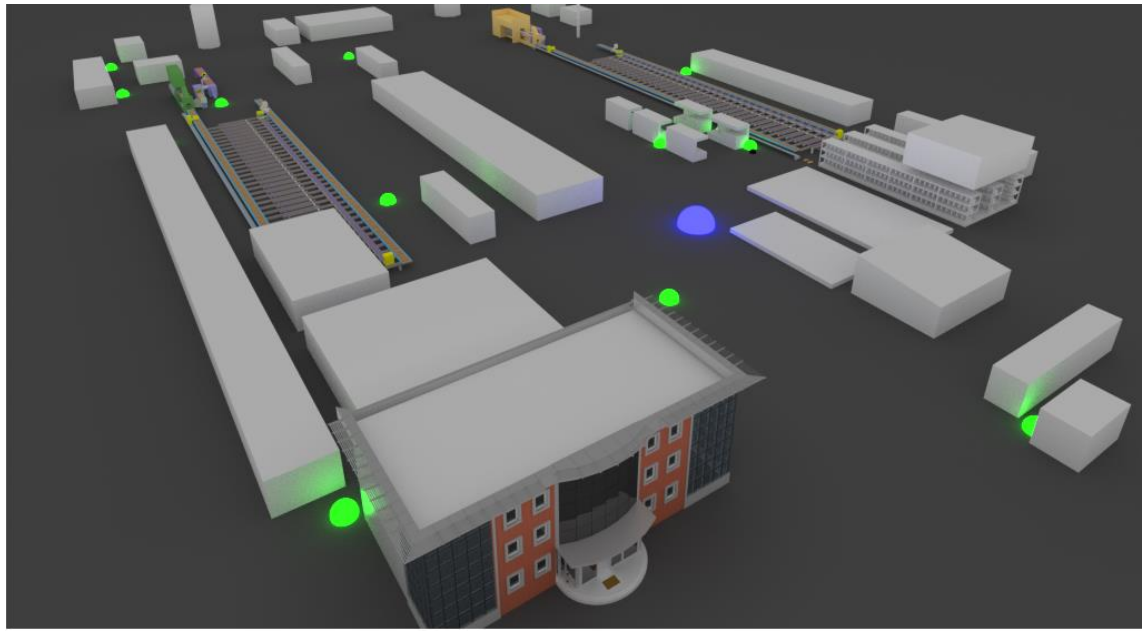


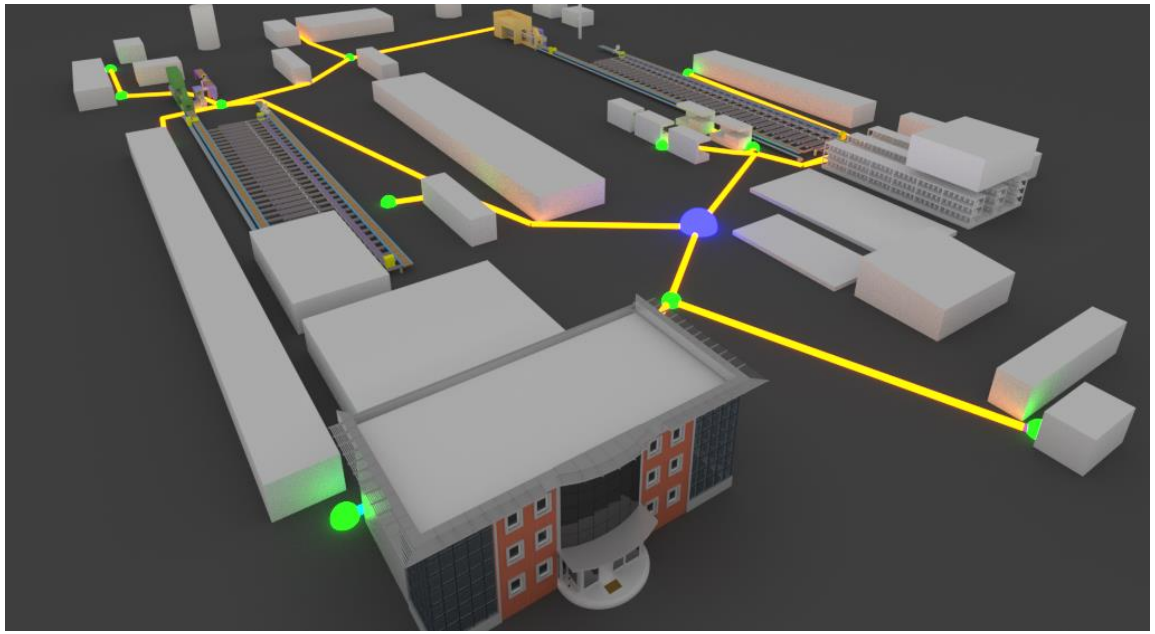


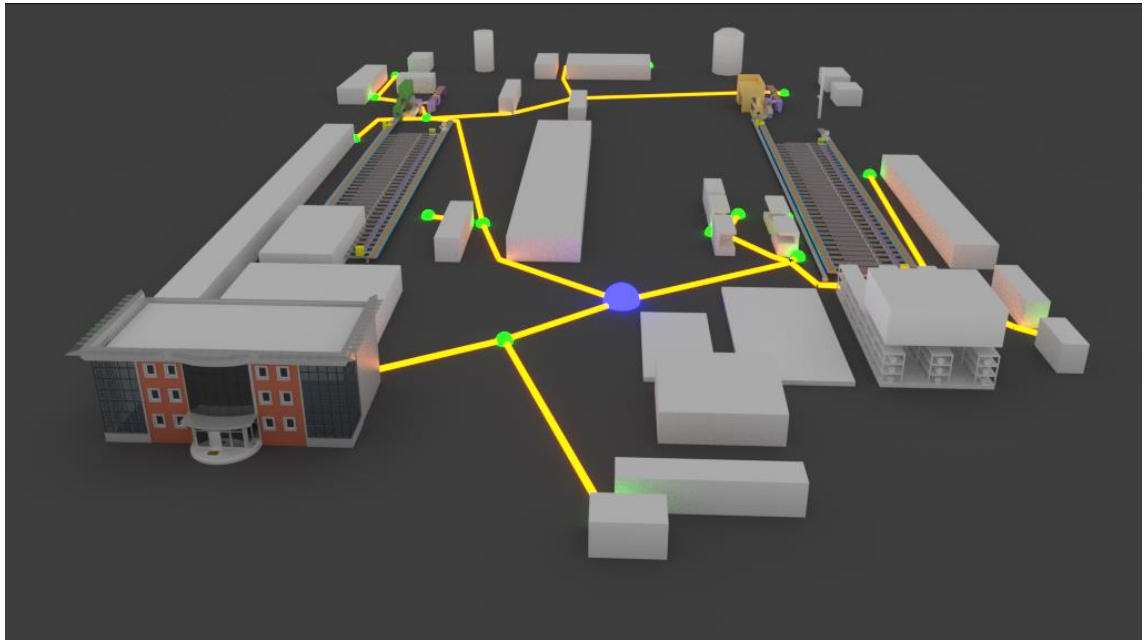

(63) factory

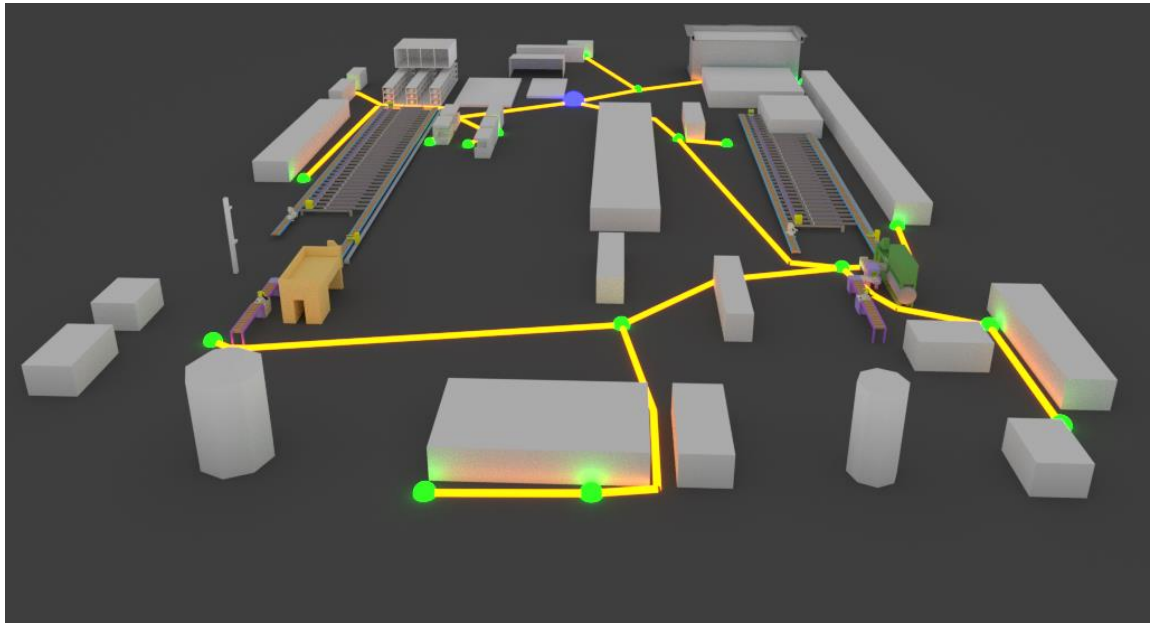

(63) factory





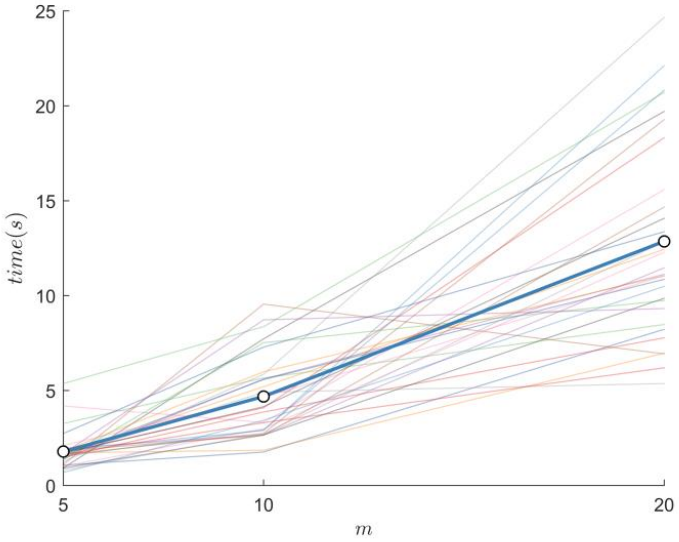




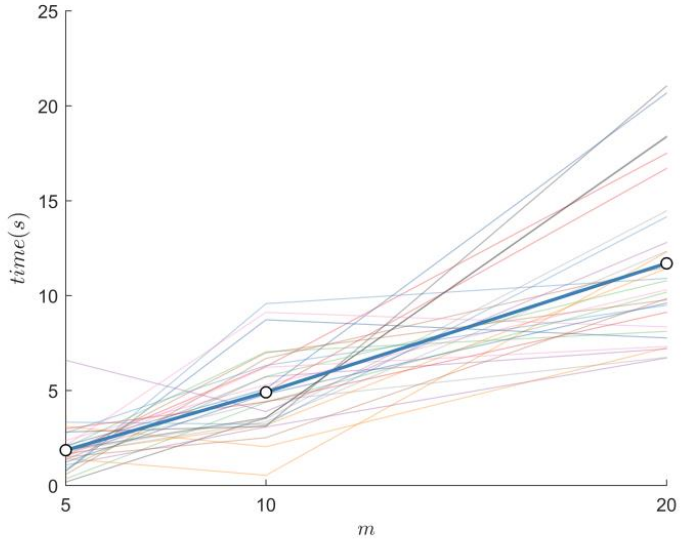


Computational Cost

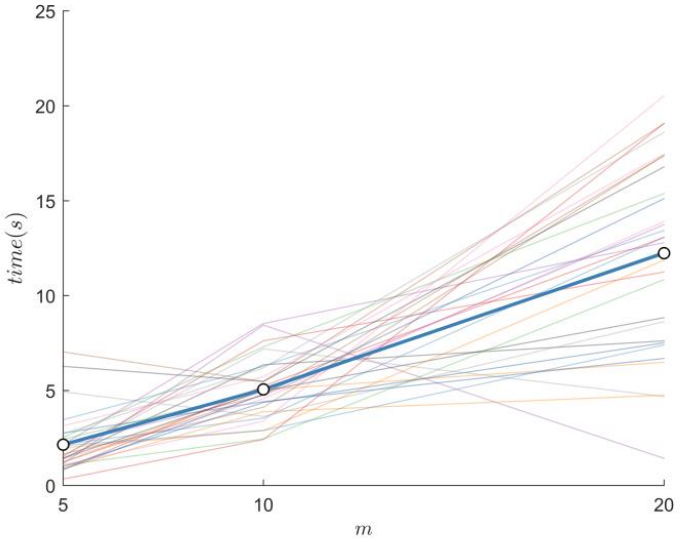
Time to generate minimal trees



(a) Map 1



(b) Map 2



(c) Map 3

Average linear behaviour: attractive scalability

Criteria for Human-Competitiveness

B Result >= recent achievement in the field

D Result is publishable in its own scientific right

G Result solves a problem of indisputable difficulty

Comparison to Human made solution

- Automated method to compute minimal trees
- Human-competitive improvement to the solution of a long-standing problem
- Challenges the local-search and the expert-picked preferences

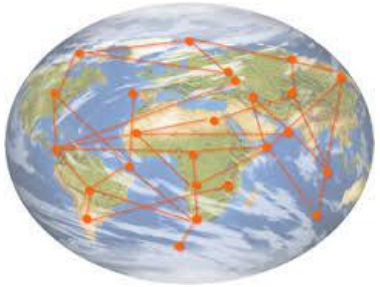
Potential Impact

Design optimal interconnecting networks

(Physical) Neural Networks

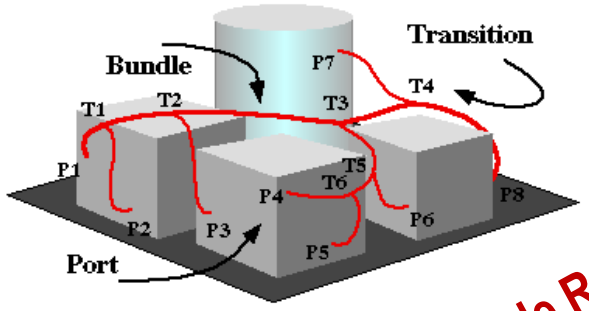
Optimal Wiring

Transportation, Distribution



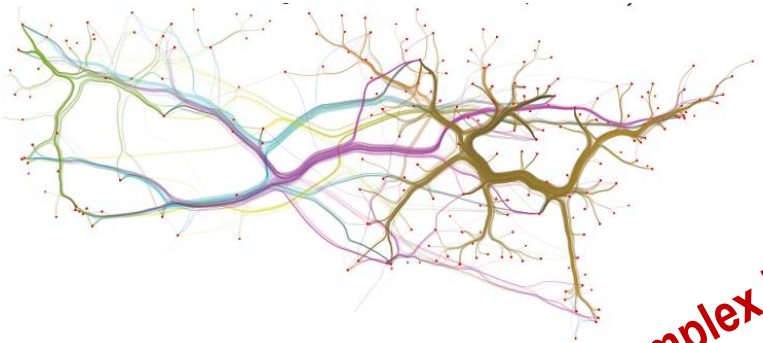
Steiner Tree

Circuit Design: VLSI



Cable Routing

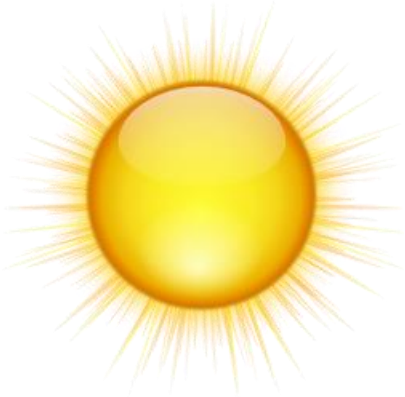
Graph Visualization



Complex Networks

Why this is the best entry?

- **Innovates the field with a novel approach to a long-standing problem.**
- **Step towards the design of large-scale minimal trees, ubiquitously.**
- **Opens a new field to tackle minimal trees based on Nature.**



Thank you