A Unified Framework of Graph-Based Evolutionary Multitasking Hyper-Heuristic

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***Abstract*—In recent research, hyper-heuristics have attracted increasing attention in various fields. The most appealing feature of hyper-heuristics is that they aim to provide more generalized solutions to optimization problems by searching in a high-level space of heuristics instead of direct problem domains. Despite the promising findings in hyper-heuristics, the design of more general search methodologies still presents a key research. Evolutionary multitasking is a relatively new evolutionary paradigm which attempts to solve multiple optimization problems simultaneously. It exploits the underlying similarities among different optimization tasks by transferring information among them, thus accelerating the optimization of all tasks. Inherently, hyper-heuristics and evolutionary multitasking are similar in the following three ways: 1) they both operate on third-party search spaces; 2) high-level search methodologies are universal; and 3) they both conduct cross-domain optimization. To integrate their advantages effectively, i.e., the knowledge-transfer and cross-domain optimization of evolutionary multitasking and the search in the heuristic spaces of hyper-heuristics, in this article, a unified framework of evolutionary multitasking graph-based hyper-heuristic (EMHH) is proposed. To assess the generality and effectiveness of the EMHH, population-based graph-based hyper-heuristics integrated with evolutionary multitasking to solve exam timetabling and graph-coloring problems, separately and simultaneously, are studied. The experimental results demonstrate the effectiveness, efficiency, and increased the generality of the proposed unified framework compared with single-tasking hyper-heuristics.**

***Index Terms*—Evolutionary multitasking, exam timetabling, graph coloring, hyper-heuristics.**

I. Introduction

META-HEURISTICS are highly effective in solving various combinational optimization problems [1], [2]. They quite often concern one particular problem, but tend to perform poorly on other problems or even other instances of the same problem. The performance of these approaches also strongly depends on domain-specific knowledge and expertise such as complicated parameter tunings [3]–[5]. Such tailor-made settings limit their generality, making them expensive to adapt to other problems.

Motivated by this, more recent research has focused on generalized and adaptive algorithms [6]. Hyper-heuristics, which are heuristics that choose heuristics, can be regarded as such general algorithms [3], [6]–[10]. Instead of searching directly in the solution space like conventional meta-heuristics, hyper-heuristics work at the higher-level search space of a set of low-level heuristics. The goal is to solve the problem at hand by selecting existing low-level heuristics or generating new low-level heuristics. The only requirement for developing a hyper-heuristic for a problem is a set of low-level heuristics that are easy-to-implement and a problem-specific objective function. These are used at the low-level in hyper-heuristics, which are general addressing different problems.

After the term was first proposed in [4], hyper-heuristic approaches have been successfully used to solve a range of combinational optimization problems, such as Boolean satisfiability problems [11], [12], vehicle routing problems [13], [14], packing problems [15], [16], educational timetabling problems [9], and many more [8], [10]. The search spaces of hyper-heuristics either comprise existing low-level heuristics or components and operators that are used to construct low-level heuristics. Based on these properties, hyper-heuristics can be categorized into heuristic selection and heuristic generation hyper-heuristics, respectively [17]. Heuristic selection hyper-heuristics select a given set of low-level heuristics to construct or improve solutions, while heuristic generation hyper-heuristics generate new heuristics using a given set of components and operators. Furthermore, in both the heuristic selection and heuristic generation hyper-heuristics, constructive and perturbative low-level heuristics can be used to build solutions step by step or to modify and improve complete solutions. This article concerns a new framework of selection constructive hyper-heuristic.

The paradigm of evolutionary multitasking optimization (EMO) was first proposed in [18] to solve multifactorial optimization (MFO) problems, which are categorized as the third category of optimization problems in addition to single-objective and multiobjective optimization problems. EMO has been successfully extended since then to several domains, including continuous optimization, discrete optimization, combinational optimization, and multiobjective optimization [18]–[25]. EMO can optimize two or more tasks

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simultaneously instead of evaluating every task at each step of the evaluation. Under the assumption that each individual is at least skilled at one task, in EMO, the population is split into different skill groups. The success of EMO lies in the transfer of knowledge among different skill groups; that is, the genetic experience within one group can be transferred to other groups, thus accelerating the convergence of all tasks [18], [24]. Moreover, the computational cost can be greatly decreased compared with that in solving all tasks separately and sequentially.

Inspired by EMO, a generalized EMO framework was proposed in [24], where the knowledge learned from computationally cheap problems is utilized to assist the optimization of computationally expensive problems via the knowledge-transfer mechanism. Li *et al.* [26] extended the evolutionary multitasking framework to a multitasking sparse reconstruction framework for solving sparse reconstruction problems. In [27], instead of performing knowledge transfer implicitly via genetic operators, a denoising autoencoder was designed to explicitly transfer the solutions among different tasks. To utilize the limited resources more efficiently, Gong *et al.* [28] deigned an online dynamic resource allocation strategy that can allocate computational resources to tasks based on their computational complexities. The literature above extends the original EMO in various aspects, including but not limited to knowledge-transfer efficiency, computational efficiency, and the applications. However, they are built on the direct solution space, which is still subject to specific problems.

This article proposes a unified framework of graph-based evolutionary multitasking hyper-heuristic (EMHH) that inherently integrates hyper-heuristics and EMO based on the three synergies between them. First, they both operate in a third-party search space rather than the direct problem-solution space. In EMO, the variables of different tasks are mapped into a unified representation, while hyper-heuristics search in a high-level space of heuristics. Second, the high-level algorithms in hyper-heuristics are equivalent to the general solvers in EMO. Third, both methodologies concern cross-domain searches, but with different mechanisms. Inspired by these synergies and similarities, the EMHH combines their advantages, namely, the knowledge transfer and the cross-domain search of EMO and the high-level search of hyper-heuristics, to further enhance the generality of the integrated approach to a higher level.

Among the different categories of hyper-heuristics, graph-based hyper-heuristics have been mostly studied to solve educational timetabling problems [9], [10]. This article focuses on the integration of EMO with population-based graph-based hyper-heuristics as the precursor to multitasking the hyper-heuristics. Carter's benchmark [29] is used to assess the effectiveness of EMHH. In particular, exam timetabling problems (ETTPs) and graph-coloring problems (GCPs) derived from Carter's benchmark, in separate and simultaneous manners, are investigated. The experimental results demonstrate that EMHH provides superior effectiveness, efficiency, and generality compared with single-tasking hyper-heuristics, especially on asynchronous optimization and synchronous cross-domain optimization. More importantly, the results show that EMHH is able to learn the implicit commonalities between solutions of different tasks in the high-level search space via knowledge-transfer mechanism, and efficiently share them across tasks to facilitate convergence of all tasks. Additionally, the knowledge-transfer mechanism in EMHH shows to improve its ability escaping from local optima.

The novelty of this article lies on the introduction of, for the first time, the concept of evolutionary multitasking into hyper-heuristics, leading to the following two contributions: first, a unified framework, i.e., EMHH, is developed which coherently integrates the two methodologies to achieve a higher level of generality. While the existing hyper-heuristics focus on handling one task at a time, which is still of limited generality, the proposed EMHH is capable of handling multiple tasks, either intradomain or cross-domain, simultaneously, thus extending the generality and scope of the general algorithms addressing multiple optimization problems. Moreover, the generality of the concept of unification in multitasking is extended, i.e., the unified representation should not be limited to the solution representation space. Other unification schemes may be promising in terms of exploiting the potential of multitasking as well. Second, this article explores the underlying communalities in the selections of heuristics for solving different problems via evolutionary multitasking with knowledge transfer in the heuristic space.

The remainder of this article is organized as follows. Section II introduces the background of graph-based hyper-heuristics and EMO, followed by the motivations. The EMHH framework is presented in Section III. Section IV presents and discusses the experimental studies on ETTPs and GCPs. The potential applications and future challenges are also provided in this section. Finally, Section V concludes this article and provides directions for future work.

II. Background

A. Graph Heuristics

Welsh and Powell [30] established the connection between timetabling and scheduling problems with graph coloring, which subsequently inspired the application of graph heuristics in solving these problems. Graph heuristics order the vertexes in a graph according to the difficulties of coloring them using feasible colors. By representing the events in timetabling problems as vertices and the edges as clashes between the events, the timetabling of events with timeslots is transferred into the problem of assigning vertices with colors. The degree of an event indicates the number of conflicting events, i.e., the events associated to it. Graph heuristics can then be used to order the events according to the difficulties of scheduling them into feasible timeslots.

The following five graph heuristics and a random heuristic have been widely used in timetabling [9], [10], [31].

1) The saturation degree (SD) indicates the number of feasible timeslots for an event. A smaller SD value indicates that fewer feasible timeslots are available for an event, thus scheduling it is more difficult than scheduling those with more available timeslots. The smallest SD heuristic selects and schedules the events with the smallest SD

values first, after which the SD values for the remaining events will be updated.
2) The largest degree (LD) heuristic selects and schedules, at each step, the event with the highest degree.
3) The largest colored degree (LCD) heuristic selects and schedules, at each step, the event with the highest number of conflicts with those scheduled in the timetable. The LCD values of the remaining unscheduled events will be updated afterward.
4) The largest weighted degree (LWD) indicates the LD weighted by the number of participants (such as students in exam timetabling) involved in the conflicting events. This heuristic selects and schedules the event with the LWD at each step.
5) The largest enrollment (LE) heuristic selects and schedules, at each step, the event with the largest number of participants involved.
6) Random ordering (RO) randomly selects and schedules an event which has not yet been scheduled.

The EMHH framework developed in this article is a selection constructive hyper-heuristic, which is defined in Definition 1 [10]. In this article, the low-level construction heuristic set H in Definition 1 comprises two of the above six constructive graph heuristics. EMHH is population-based, i.e., it evolves on a set of low-level heuristic sequences in a population.

Definition 1: Given a problem β and a set of low-level construction heuristics $H = \{h_0, h_1, \ldots, h_n\}$ for the problem domain, a selection constructive hyper-heuristic constructs a complete solution s for β from its initial state s_0 to the final state s by repeatedly selecting and applying a low-level heuristic from H to change from one solution state s_i to the next s_{i+1}.

B. Evolutionary Multitasking Optimization

EMO is a new branch of evolutionary algorithms (EAs) [18] to address the MFO problems defined in (1)

$$\{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_K\} = \text{argmin} \ \{f_1(\boldsymbol{x}_1), f_2(\boldsymbol{x}_2), \ldots, f_K(\boldsymbol{x}_K)\} \quad \text{s.t.} \ \boldsymbol{x}_k \in \Omega_k, k \in \{1, 2, \ldots, K\} \quad (1)$$

where K is the number of tasks involved. $\boldsymbol{x}_k = (x_{k,1}, x_{k,2}, \ldots, x_{k,D^k})$ represents a feasible solution for the kth task T_k, f_k is its objective function, D_k denotes its dimensionality, and Ω_k is the search space, respectively.

Compared to multiobjective optimization, there are two major distinctive features in MFO. First, no dependence exists between tasks, i.e., no prior knowledge on the relationship among the K tasks needs to be identified beforehand, while the tasks in multiobjective optimization explicitly conflict with each other. Second, the variable spaces of K tasks are heterogeneous, namely, each task is evaluated in its own design space.

To compare the individuals in the population and clarify the relationships between individuals and tasks, the following definitions are proposed in [18] for EMO.

1) *Factorial Rank:* The factorial rank r_k^i of individual p_i for the kth task is the index of p_i sorted in an ascending order by the objective function value f_k of the kth task.
2) *Skill Factor:* The skill factor τ_i of individual p_i indicates the task that p_i is most skillful in, i.e., $\tau_i = \text{argmin}_k\{r_k^i\}$, $k \in \{1, 2, \ldots, K\}$.
3) *Scalar Fitness:* The scalar fitness ϕ_i of individual p_i is calculated as $\phi_i = 1/\min\{r_k^i\}$, $k \in \{1, 2, \ldots, K\}$.

C. Motivations

Although hyper-heuristics search in heuristics space, their current paradigms still focus on solving isolated problems or isolated problem domains independently. However, as stated in [20], *real-world problems seldom exist in isolation*, thus the potential of hyper-heuristics might be underestimated. EMO shown to perform well in solving MFO problems based on the unified representation and the knowledge-transfer mechanism [18]–[28]. It also generalize well in handling multiple optimization tasks simultaneously.

In addition, the direct solutions of different problems usually have no common structures or connection with each other, e.g., the 2-D timetable compared to the 1-D coloring of a graph. However, the solutions of these two problems in the heuristic space may share similar patterns. For example, in [32], it is found that LWD rather than SD tends to generate better solutions at the early stage of solution construction for ETTPs.

Given the above facts, this article proposes an efficient and more general hyper-heuristic framework by building the evolutionary multitasking in the heuristic space. To be specific, on the one hand, this article combines evolutionary multitasking with hyper-heuristics to raise the generality of both of them to a higher level, namely, endow hyper-heuristics the ability to handle multitasking problems, besides, extend the concept of the unification scheme in evolutionary multitasking to the heuristic space. Moreover, the integration of unification schemes into hyper-heuristics may be adopted in other hyper-heuristic approaches, and the same unification schemes in evolutionary multitasking may be applied to solving different problems. On the other hand, the proposed framework aims to investigate the underlying similar patterns among different tasks in the heuristic space based on the concept of evolutionary multitasking and knowledge transfer to facilitate effective convergence of optimization.

III. Evolutionary Multitasking Hyper-Heuristic Framework

A. Framework of EMHH

In the unified framework of EMHH shown in Algorithm 1, each individual is associated with one of the K tasks in the population to address them simultaneously. In the initial population on the low-level heuristic space, every individual is evaluated on all tasks initially, according to lines 2–7 in Algorithm 1. Then, in line 8, the factorial ranks and skill factors are calculated and assigned to all individuals. In the following generations, individual p_i will only be evaluated on the task indicated by its skill factor τ_i. The fitness values of all other unevaluated tasks are set to infinite values (i.e., a large enough number). The offspring in the unified representation, thus, inherit the skill factors from their parents, and the genetic materials of one task can be transferred to address other tasks

Algorithm 1 EMHH

Input:

N: Population size.
K: The number of tasks.
H : A set of low-level construction heuristics.

Output:

The best solutions of all tasks.

1: Initialize population $\boldsymbol{P}$ with randomly generated N low-level heuristic sequences composed from H.
2: **for** $i = 1$ to N **do**
3: **for** $k = 1$ to K **do**
4: Construct solution $s_{i,k}$ by heuristic sequence p_i for task k.
5: Evaluate $s_{i,k}$ for task k.
6: **end for**
7: **end for**
8: Initialize skill factor τ_i of p_i, $i \in \{1, 2, \ldots, N\}$. // see Section II-B
9: **while** termination criteria are not satisfied **do**
10: Generate offspring population $\boldsymbol{P}'$ using the cross-task mating operator on $\boldsymbol{P}$. // see Algorithm 2
11: Evaluate every individual in $\boldsymbol{P}'$ on the task indicated by its skill factor.
12: $\boldsymbol{R} = \boldsymbol{P} \cup \boldsymbol{P}'$.
13: Update scalar fitness of every individual in $\boldsymbol{R}$.
14: Select the N fittest individuals from $\boldsymbol{R}$ according to the scalar fitness as the next generation $\boldsymbol{P}$.
15: **end while**
16: Output the best solution obtained.

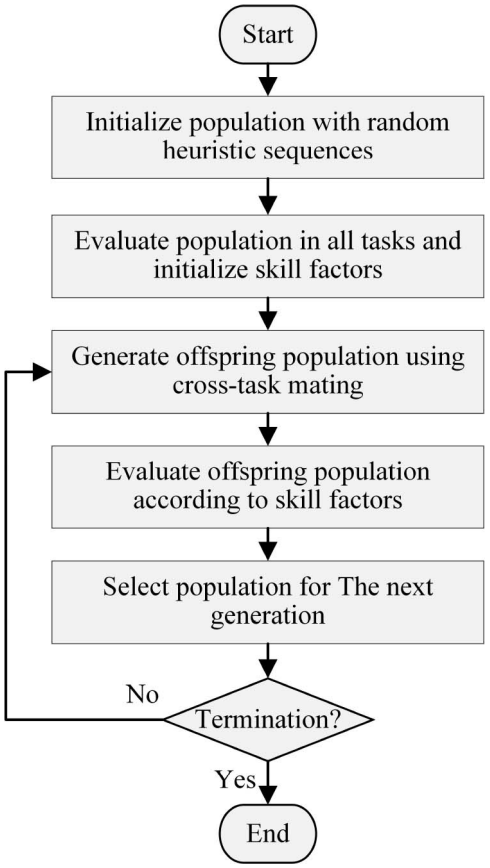

Fig. 1. Workflow of EMHH.

Algorithm 2 Cross-Task Mating

Input:

p_a, p_b: Randomly selected parents.
rmp: Predefined random mating probability.

Output:

offspring o_1, o_2.

1: **if** $\tau_a = \tau_b$ **then**
2: Crossover and mutation are applied sequentially on p_a and p_b to generate o_1 and o_2.
3: $\tau_1 = \tau_2 = \tau_a$(or τ_b).
4: **else**
5: **if** $rand < rmp$ **then**
6: Crossover and mutation are applied sequentially on p_a and p_b to generate o_1 and o_2.
7: Randomly assign τ_a or τ_b to τ_1 and τ_2.
8: **else**
9: Mutate p_a to generate o_1, $\tau_1 = \tau_a$.
10: Mutate p_b to generate o_2, $\tau_2 = \tau_b$.
11: **end if**
12: **end if**

during the evolution. The workflow of EMHH is presented in Fig. 1.

In EMHH, a cross-task mating operator is used to generate the offspring, as shown in Algorithm 2. This follows the same paradigm as that used in [26]. In Algorithm 2, if the selected parents have identical skill factors such as in case 1 in Fig. 2, the offspring generated by the crossover and mutation operators inherent the same skill factors as those of their parents. Otherwise, the transfer of genetic materials between parents depending on a predefined probability rmp, as shown in lines 5–11 in Algorithm 2, is conducted. As seen from Fig. 2, the knowledge transfer among tasks occurs in case 3. Since the selected parents have different skill factors, each of their offspring has two alternative skill factors to inherit. This eventually results in four possible combinations of offspring, as shown in the dashed rectangle of case 3, where all have genetic materials transferred. For example, assume o_1 and o_2 are derived from p_a and p_b, respectively, then in the first combination, o_2 inherits p_a's skill factor rather than p_b's, and it will be evaluated on the task indicated by the skill factor of p_a. Consequently, the genetic materials learned in optimizing p_b's task are transferred to p_a, which will likely be helpful in transferring the meaningful building-blocks from one task to the others.

B. Solution Representation and Evaluation

The chromosome of the individual is represented as a sequence of heuristics. Recall that the dimensionalities of the direct solutions for the K tasks could be different. The length of individual heuristic sequences is denoted by $D^{\max} = \max\{D^k\}$, $k \in \{1, 2, \ldots K\}$. Fig. 3 shows an example of how

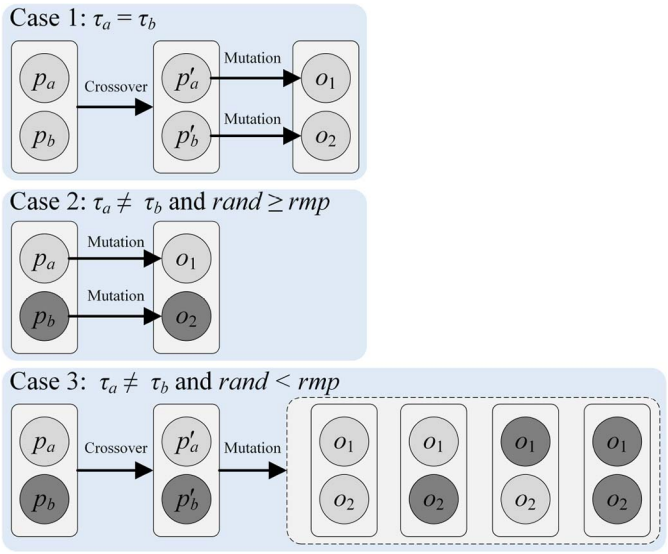

Fig. 2. Process of cross-task mating. The light and dark grayed cycles represent different skill factors.

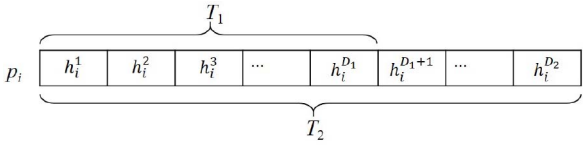

Fig. 3. Example of solution representation.

the chromosome of two tasks T_1 and T_2 with the dimensionalities of D_1 and D_2, respectively, where $D_1 < D_2$, are encoded. Each bit in Fig. 3 represents a low-level heuristic. If the individual p_i in Fig. 3 is associated with task T_1, that is, the skill factor of p_i is $\tau_i = 1$, then, D_1 heuristics selected from the beginning of p_i are employed to construct the solution for T_1, while the remaining $D_2 - D_1$ heuristics are discarded. Otherwise, if $\tau_i = 2$, all the heuristics are employed to construct the solution for T_2. The representation for problems with more than two tasks follows the same scheme. In cases where the prior experience is unavailable, the representations for different tasks all start from the beginning of the chromosome by default [26].

The heuristic sequence p_i itself cannot be evaluated directly without associating it with a specific task or problem; that is, the fitness of a heuristic sequence depends on the quality of the solution constructed by p_i. For example, in the ETTP, every heuristic in the heuristic sequence is used to schedule one or several exams. The heuristic is scanned one by one in the heuristic sequence p_i. At each construction step, the first unused heuristic h_i in the heuristic sequence p_i is employed to select and then schedule the exams. After a complete timetable is constructed, its soft constraint penalty cost is calculated and used as the fitness of the heuristic sequence p_i.

IV. Study on Exam Timetabling and Graph-Coloring Problems

In this section, the performance of EMHH is evaluated on two to five-task intra and cross-domain MFO problems generated by 16 Carter exam timetabling benchmarks and their corresponding GCP variants. The effectiveness, efficiency, and generality of EMHH for solving both intradomain and cross-domain problems are analyzed. Note that a task in EMHH represents an instance of ETTPs or GCPs in the following context unless otherwise stated.

A. Test Problems

ETTPs involve assigning a given number of exams to a set of predefined timeslots subject to certain constraints [7]. The constraints can be classified into two categories, namely, hard and soft constraints. The solutions that satisfy all hard constraints are called feasible solutions. Soft constraints are generally used as criteria in evaluating the quality of feasible solutions. In real cases, both hard and soft constraints vary from institution to institution. The most common constraints in ETTPs are listed as follows.

1) *Student Conflict:* Students cannot have more than one exam in one timeslot (hard constraint).
2) *Room Capacity:* The total number of students assigned cannot exceed the room capacity in one timeslot or session (hard constraint).
3) *Exam Distribution:* The exams of one student should be as sparsely distributed as possible in the timetable (soft constraint).

Three variants of ETTPs, namely, the GCP, uncapacitated ETTPs (uETTPs), and capacitated ETTPs (cETTPs), have been studied in the existing literature.

1) GCPs aim to find the smallest number of colors for all vertices without conflicts; that is, adjacent vertices are assigned different colors. An ETTP degrades to a GCP if only student conflict is taken into consideration [7], [33].
2) In uETTPs, the room capacity constraint is relaxed. A penalty $w_t = 2^{5-t}, t \in [1, 5]$ occurs if two exams of a student are assigned t timeslots apart [7]. The objective is to minimize the soft constraint penalty cost ϕ = Total_penalty/Number_of_students, where Total_penalty is the summation of the penalties caused by all students. This objective represents a preference to timetables where each student's exams are distributed as sparsely as possible.
3) In contrast to uETTPs, in cETTPs, room capacity restrictions apply per timeslot [34], [35] or per session [33]. The objective of cETTPs is to minimize the number of students taking two exams consecutively during one day [34], and it can also be extended to overnight cases [33], [35].

In this article, we employ version I, II and IIc (the corrected version II) of the Carter benchmarks summarized in [7] as the uETTP and GCP test sets. A brief summary of the properties of these benchmarks can be found at http://www.cs.nott.ac.uk/pszrq/data.htm and in Table SI in the supplementary material. For uETTPs, instead of using the above-mentioned penalty w_t, a modified penalty w'_t, as shown in (2), is adopted to calculate Total_penalty in this article, where a large penalty occurs for each pair of conflicting exams. The resulting search space of the proposed algorithm thus

Algorithm 3 Adaptive One-Point Mutation

Input:
p: Parent heuristic sequence.
gen: Current generation.
G: Total number of generations.
Output:
o: Offspring.
$mLength = max(1, |p| \cdot gen/G)$. // Decide the mutation part in p, i.e. $p(1) \sim p(mLength)$.
$pm = 1/mLength$. // Probability of mutation.
$o = p$.
for each position $i = 1$ to $mLength$ **do**
  if $rand < pm$ **then**
    //$rand \in (0, 1)$ is a random real number.
    randomly change $o(i)$ to another heuristic.
  end if
end for
return o.

contains infeasible solutions; however, the fitness landscape becomes connected. The greedy strategy in [5] and [32] is used to choose the timeslots for the selected exams

$$w'_t = \begin{cases} 2^{5-t}, & t \in [1, 5] \\ 10\,000, & t = 0. \end{cases} \tag{2}$$

For the GCPs, the direct use of the number of colors in the objective function is very likely to form a fitness landscape with large plateaus; thus, it is difficult to distinguish between different solutions with the same number of colors. In this article, we modified the evaluation function in [5] to include two measures on the coloring, namely, the coloring sum as used in [5] and the cube of the number of colors used. The coloring sum is calculated as the sum of the product of the size and the color index of each color class that comprises a set of vertices with the same color. Thus, this new evaluation function considers not only the number of colors but also the sizes of the color classes.

B. Genetic Operators

In [32] and [36], it is shown that the heuristics at the later stage of solution construction tend to make less difference in the quality of the solutions. To dynamically allocate the computational resources in EMHH, the adaptive one-point mutation operator shown in Algorithm 3 is used to first modify early parts of the heuristic sequences and then extend to later parts along with the evolution. $|p|$ represents the length of individual p. As a result, the earlier heuristics in the heuristic sequences have more opportunities to evolve toward proper hybridizations. The uniform crossover [37] is used in EMHH.

C. Experimental Setup

In this article, only the SD and LWD are employed as the candidate low-level heuristics in EMHH. According to [29], [31], and [38], when used alone, SD outperforms the other graph heuristics, and was shown to be effective in [32] when hybridized with LWD. Each of the graph heuristics will schedule two events, as in [31], to construct the solutions.

In EMHH, the population size is set to 30 and the total number of generations is set to 100. The single-tasking optimization framework, termed as SOHH, is developed with every setting identical to EMHH, except that only one task is optimized.

The predefined probability *rmp* controls how often the evolved knowledge is transferred among different tasks in EMHH. As suggested in [25], *rmp* should be close to 1 if there exists prior knowledge that the handling tasks are correlated; otherwise, a smaller *rmp* value should be set. In the literature, no effective measurement between uETTP instances exists due to the difficulty in assessing their similarity. In a set of preliminary experiments, the range of *rmp* is set to [0.1, 1], with an increasing stepsize of 0.1, to examine the impact of *rmp* on the performance of EMHH for solving three MFO problems (hec92, hec92 II), (hec92, sta83), and (hec92, tre92). The average results from 30 independent runs, as plotted in Fig. 4, show that EMHH performed relatively well on (hec92, hec92 II) and (hec92, tre92) with $rmp = 0.8$, and the performance on (hec92, sta83) is less sensitive to the settings of *rmp*. We, therefore, set *rmp* to 0.8 in the remaining experiments.

All the algorithms were implemented in C++ using Visual Studio 2017. The experiments were conducted on a desktop with Intel Core i7-3820 CPU (3.60 GHz) 16.0-GB memory and 64-bit Windows 10. The average results are obtained over 30 independent runs of the algorithms.

D. Experimental Results

An additional five sets of experiments were conducted to evaluate the EMHH framework. The first three experiments demonstrate the effectiveness, efficiency, and generality of asynchronous optimization of EMHH for intradomain problems. The fourth set of experiments presents the generality of EMHH in addressing cross-domain optimization problems. The generality of EMHH is further examined on problems with over two tasks in the fifth set of the experiment. Finally, comparisons between EMHH and other existing hyper-heuristics are presented.

1) Comparison Between EMHH and SOHH on uETTPs: First, we set K to 2; thus, in total, 120 MFO problems are created from the 16 Carter exam timetabling benchmarks. The best, average, and standard deviations of objective values obtained by EMHH and SOHH are presented in Table I, where T_1 and T_2 represent task 1 and task 2 in an MFO problem, respectively. For example, in the first MFO problem (car91, car92), T_1 and T_2 represent car91 and car92, respectively. The Wilcoxon rank-sum test with a 95% confidence level is conducted between the results of EMHH and SOHH. In Table I, the significantly better results are highlighted in gray, and the better results are in light gray. The detailed p and h values are provided in Table SIII in the supplementary material, where $h = 1$ or $p < 0.05$ denotes a significantly better performance at the confidence level of 95%. Table II compares the EMHH and SOHH results listed in Table I using three indicators, which are defined as follows.

1) *All-Win:* If the results on both tasks, i.e., T_1 and T_2, in an MFO problem obtained by approach A are all

TABLE I
MINIMUM, MEAN, AND STANDARD DEVIATIONS OBTAINED BY EMHH AND SOHH ON THE UETTPS

MFOs	EMHH		SOHH	
	T_1	T_2	T_1	T_2
car91	5.14	4.27	5.18	4.31
car92	5.22	4.39	5.26	4.41
	0.04	0.04	0.04	0.04
car91	5.15	36.07	5.18	36.62
ear83	5.22	36.98	5.26	37.12
	0.03	0.51	0.04	0.25
car91	5.15	39.21	5.18	39.51
ear83 IIc	5.23	40.74	5.26	41.13
	0.03	0.93	0.04	1
car91	5.19	11.66	5.18	11.69
hec92	5.22	11.95	5.26	12.12
	0.03	0.15	0.04	0.267
car91	5.11	11.62	5.18	11.67
hec92 II	5.22	11.97	5.26	12.1
	0.04	0.19	0.04	0.29
car91	5.16	15.07	5.18	15.37
kfu93	5.23	15.52	5.26	15.68
	0.03	0.21	0.04	0.15
car91	5.14	11.23	5.18	11.65
lse91	5.23	11.69	5.26	11.85
	0.03	0.23	0.04	0.16
car91	5.18	9.4	5.18	9.34
rye93	5.23	9.57	5.26	9.55
	0.03	0.08	0.04	0.08
car91	5.19	158.66	5.18	169.77
sta83	5.23	159.31	5.26	169.86
	0.03	0.48	0.04	0.09
car91	5.16	34.24	5.18	34.26
sta83 IIc	5.23	34.65	5.26	34.99
	0.03	0.19	0.04	0.22
car91	5.17	8.61	5.18	8.78
tre92	5.24	8.8	5.26	8.88
	0.03	0.09	0.04	0.06
car91	5.12	3.39	5.18	3.39
uta92	5.22	3.42	5.26	3.43
	0.04	0.02	0.04	0.02
car91	5.17	3.38	5.18	3.35
uta92 II	5.23	3.41	5.26	3.41
	0.03	0.02	0.04	0.03
car91	5.16	27.66	5.18	28.31
ute92	5.22	28.25	5.26	28.75
	0.04	0.3	0.04	0.14
car91	5.18	41.06	5.18	41.05
yor83	5.23	41.98	5.26	42.87
	0.03	0.43	0.04	2.91
car92	4.32	35.5	4.31	36.62
ear83	4.4	36.62	4.41	37.12
	0.03	0.48	0.04	0.25
car92	4.3	38.99	5.18	39.51
ear83 IIc	4.38	40.74	5.26	41.13
	0.04	1.25	0.04	1
car92	4.33	11.57	4.31	11.69
hec92	4.39	11.94	4.41	12.12
	0.03	0.22	0.04	0.27
car92	4.35	11.63	4.31	11.67
hec92 II	4.39	11.93	4.41	12.1
	0.03	0.2	0.04	0.29
car92	4.31	15.2	4.31	15.37
kfu93	4.39	15.6	4.41	15.68
	0.04	0.16	0.04	0.15
car92	4.34	11.3	4.31	11.65
lse91	4.39	11.62	4.41	11.85
	0.03	0.13	0.04	0.16
car92	4.35	9.43	4.31	9.34
rye93	4.39	9.61	4.41	9.55
	0.03	0.08	0.04	0.08
car92	4.35	158.56	4.31	169.77
sta83	4.4	159.33	4.41	169.86
	0.03	0.35	0.04	0.09
car92	4.32	34.3	4.31	34.26
sta83 IIc	4.38	34.65	4.41	34.99
	0.03	0.17	0.04	0.22
car92	4.29	8.63	4.31	8.78
tre92	4.39	8.8	4.41	8.88
	0.04	0.08	0.04	0.06
car92	4.32	3.39	4.31	3.39
uta92	4.38	3.43	4.41	3.43
	0.03	0.02	0.04	0.02
car92	4.32	3.37	4.31	3.35
uta92 II	4.38	3.41	4.41	3.41
	0.04	0.02	0.04	0.03
car92	4.33	27.69	4.31	28.31
ute92	4.39	28.29	4.41	28.75
	0.04	0.34	0.04	0.14
car92	4.33	40.99	4.31	41.05
yor83	4.39	41.86	4.41	42.87
	0.03	0.43	0.04	2.91
ear83	35.44	39.38	36.62	39.51
ear83 IIc	36.58	40.11	37.12	41.13
	0.56	0.62	0.25	1
ear83	35.88	11.68	36.62	11.69
hec92	36.75	11.99	37.12	12.12
	0.54	0.16	0.25	0.27
ear83	35.79	11.63	36.62	11.67
hec92 II	36.64	12.01	37.12	12.1
	0.4	0.2	0.25	0.29
ear83	35.9	15.09	36.62	15.37
kfu93	36.75	15.57	37.12	15.68
	0.5	0.16	0.25	0.15
ear83	35.49	11.07	36.62	11.65
lse91	36.84	11.7	37.12	11.85
	0.57	0.18	0.25	0.16
ear83	35.68	9.43	36.62	9.34
rye93	36.8	9.57	37.12	9.55
	0.51	0.08	0.25	0.08
ear83	36.01	158.56	36.62	169.77
sta83	36.91	159.5	37.12	169.86
	0.54	0.49	0.25	0.09
ear83	35.9	34.06	36.62	34.26
sta83 IIc	36.84	34.64	37.12	34.99
	0.44	0.23	0.25	0.22
ear83	35.31	8.62	36.62	8.78
tre92	36.78	8.81	37.12	8.88
	0.48	0.08	0.25	0.06
ear83	35.64	3.38	36.62	3.39
uta92	36.81	3.42	37.12	3.43
	0.38	0.02	0.25	0.02
ear83	35.71	3.38	36.62	3.35
uta92 II	36.79	3.41	37.12	3.41
	0.52	0.02	0.25	0.03
ear83	36.19	27.23	36.62	28.31
ute92	36.77	28.31	37.12	28.75
	0.33	0.35	0.25	0.14
ear83	35.82	40.65	36.62	41.05
yor83	36.65	41.78	37.12	42.87
	0.49	0.62	0.25	2.91
ear83 IIc	39.58	11.56	39.51	11.69
hec92	40.71	11.96	41.13	12.12
	0.78	0.21	1	0.27
ear83 IIc	39.41	11.63	39.51	11.67
hec92 II	40.47	11.99	41.13	12.1
	0.77	0.22	1	0.29
ear83 IIc	39.41	14.99	39.51	15.37
kfu93	40.42	15.58	41.13	15.68
	0.67	0.18	1	0.15
ear83 IIc	39.36	11.4	39.51	11.65
lse91	40.87	11.68	41.13	11.85
	2.65	0.15	1	0.16
ear83 IIc	39.37	9.31	39.51	9.34
rye93	40.65	9.56	41.13	9.55
	1.17	0.09	1	0.08
ear83 IIc	39.51	158.71	39.51	169.77
sta83	41.31	159.41	41.13	169.86
	2.44	0.38	1	0.09
ear83 IIc	39.25	34.43	39.51	34.26
sta83 IIc	40.16	34.7	41.13	34.99
	0.61	0.17	1	0.22
ear83 IIc	39.29	8.69	39.51	8.78
tre92	40.78	8.8	41.13	8.88
	1.04	0.06	1	0.06
ear83 IIc	38.94	3.39	39.51	3.39
uta92	40.22	3.43	41.13	3.43
	0.71	0.02	1	0.02
ear83 IIc	39.52	3.36	39.51	3.35
uta92 II	40.4	3.41	41.13	3.41
	0.68	0.02	1	0.03
ear83 IIc	39.03	27.64	39.51	28.31
ute92	40.47	28.37	41.13	28.75
	0.71	0.3	1	0.14
ear83 IIc	39.39	40.16	39.51	41.05
yor83	40.46	41.78	41.13	42.87
	0.59	0.6	1	2.91
hec92	11.5	11.51	11.69	11.67
hec92 II	11.91	11.95	12.12	12.1
	0.21	0.19	0.27	0.29
hec92	11.69	15.05	11.69	15.37
kfu93	11.99	15.54	12.12	15.68
	0.2	0.17	0.27	0.15
hec92	11.68	11.29	11.69	11.65
lse91	12.05	11.61	12.12	11.85
	0.22	0.16	0.27	0.16
hec92	11.59	9.36	11.69	9.34
rye93	11.99	9.59	12.12	9.55
	0.22	0.1	0.27	0.08
hec92	11.49	158.61	11.69	169.77
sta83	12.07	159.35	12.12	169.86
	0.27	0.46	0.27	0.09
hec92	11.45	34.28	11.69	34.26
sta83 IIc	12.05	34.72	12.12	34.99
	0.21	0.2	0.27	0.22
hec92	11.65	8.55	11.69	8.78
tre92	11.98	8.79	12.12	8.88
	0.18	0.08	0.27	0.06
hec92	11.5	3.38	11.69	3.39
uta92	11.93	3.43	12.12	3.43
	0.28	0.02	0.27	0.02
hec92	11.63	3.37	11.69	3.35
uta92 II	11.99	3.41	12.12	3.41
	0.23	0.02	0.27	0.03
hec92	11.55	27.9	11.69	28.31
ute92	12.02	28.4	12.12	28.75
	0.26	0.24	0.27	0.14
hec92	11.6	40.21	11.69	41.05
yor83	12.01	41.74	12.12	42.87
	0.22	0.55	0.27	2.91
hec92 II	11.58	15.22	11.67	15.37
kfu93	12.02	15.6	12.1	15.68
	0.21	0.16	0.29	0.15
hec92 II	11.62	11.18	11.67	11.65
lse91	11.98	11.57	12.1	11.85
	0.21	0.2	0.29	0.16
hec92 II	11.5	9.35	11.67	9.34
rye93	11.99	9.59	12.1	9.55
	0.23	0.09	0.29	0.08
hec92 II	11.63	158.57	11.67	169.77
sta83	11.95	159.33	12.1	169.86
	0.19	0.42	0.29	0.09
hec92 II	11.63	33.81	11.67	34.26
sta83 IIc	12.06	34.61	12.1	34.99
	0.23	0.26	0.29	0.22
hec92 II	11.53	8.62	11.67	8.78
tre92	11.98	8.83	12.1	8.88
	0.17	0.07	0.29	0.06
hec92 II	11.38	3.39	11.67	3.39
uta92	11.99	3.43	12.1	3.43
	0.24	0.02	0.29	0.02
hec92 II	11.6	3.38	11.67	3.35
uta92 II	11.95	3.42	12.1	3.41
	0.2	0.02	0.29	0.03
hec92 II	11.48	27.42	11.67	28.31
ute92	11.99	28.32	12.1	28.75
	0.21	0.34	0.29	0.14
hec92 II	11.58	40.52	11.67	41.05
yor83	11.96	41.85	12.1	42.87
	0.16	0.5	0.29	2.91
kfu93	14.79	11.24	15.37	11.65
lse91	15.54	11.62	15.68	11.85
	0.24	0.17	0.15	0.16
kfu93	15.1	9.44	15.37	9.34
rye93	15.58	9.59	15.68	9.55
	0.19	0.07	0.15	0.08
kfu93	15.23	158.53	15.37	169.77
sta83	15.57	159.38	15.68	169.86
	0.18	0.32	0.15	0.09
kfu93	15.31	34.2	15.37	34.26
sta83 IIc	15.58	34.63	15.68	34.99
	0.16	0.2	0.15	0.22
kfu93	15.37	8.6	15.37	8.78
tre92	15.6	8.81	15.68	8.88
	0.14	0.1	0.15	0.06
kfu93	15.13	3.39	15.37	3.39
uta92	15.59	3.42	15.68	3.43
	0.17	0.02	0.15	0.02
kfu93	15.26	3.37	15.37	3.35
uta92 II	15.59	3.41	15.68	3.41
	0.14	0.02	0.15	0.03
kfu93	15.23	27.74	15.37	28.31
ute92	15.6	28.41	15.68	28.75
	0.15	0.31	0.15	0.14
kfu93	15.08	41.11	15.37	41.05
yor83	15.57	41.85	15.68	42.87
	0.19	0.45	0.15	2.91
lse91	11.3	9.44	11.65	9.34
rye93	11.63	9.57	11.85	9.55
	0.2	0.08	0.16	0.08
lse91	11.2	158.52	11.65	169.77
sta83	11.56	159.29	11.85	169.86
	0.17	0.39	0.16	0.09
lse91	11.21	34.14	11.65	34.26
sta83 IIc	11.65	34.68	11.85	34.99
	0.18	0.18	0.16	0.22
lse91	11.19	8.64	11.65	8.78
tre92	11.06	8.08	11.85	8.88
	0.16	0.07	0.16	0.06
lse91	11.33	3.38	11.65	3.39
uta92	11.66	3.43	11.85	3.43
	0.2	0.03	0.16	0.03
lse91	11.31	3.34	11.65	3.35
uta92 II	11.67	3.41	11.85	3.41
	0.2	0.03	0.16	0.03
lse91	11.39	27.31	11.65	28.31
ute92	11.68	28.31	11.85	28.75
	0.16	0.31	0.16	0.14
lse91	11.18	39.65	11.65	41.05
yor83	11.68	41.8	11.85	42.87
	0.19	0.71	0.16	2.91
rye93	9.32	158.83	9.34	169.77
sta83	9.57	159.36	9.55	169.86
	0.09	0.38	0.08	0.09
rye93	9.44	34.25	9.34	34.26
sta83 IIc	9.6	34.65	9.55	34.99
	0.08	0.18	0.08	0.22
rye93	9.3	8.61	9.34	8.78
tre92	9.59	8.78	9.55	8.88
	0.09	0.08	0.08	0.06
rye93	9.43	3.4	9.34	3.39
uta92	9.57	3.43	9.55	3.43
	0.08	0.01	0.08	0.02
rye93	9.45	3.37	9.34	3.35
uta92 II	9.6	3.41	9.55	3.41
	0.08	0.02	0.08	0.03
rye93	9.37	27.8	9.34	28.31
ute92	9.59	28.42	9.55	28.75
	0.09	0.24	0.08	0.14
rye93	9.44	41.03	9.34	41.05
yor83	9.58	42.33	9.55	42.87
	0.07	2.04	0.08	2.91
sta83	158.68	34.37	169.77	34.26
sta83 IIc	159.3	34.74	169.86	34.99
	0.29	0.17	0.09	0.22
sta83	158.5	8.62	169.77	8.78
tre92	159.24	8.8	169.86	8.88
	0.39	0.06	0.09	0.06
sta83	158.59	3.4	169.77	3.39
uta92	159.36	3.43	169.86	3.43
	0.34	0.02	0.09	0.02
sta83	158.77	3.37	169.77	3.35
uta92 II	159.26	3.42	169.86	3.41
	0.4	0.02	0.09	0.03
sta83	158.6	27.26	169.77	28.31
ute92	159.17	28.15	169.86	28.75
	0.38	0.35	0.09	0.14
sta83	158.73	40.71	169.77	41.05
yor83	159.42	41.93	169.86	42.87
	0.41	0.65	0.09	2.91
sta83 IIc	34.05	8.66	34.26	8.78
tre92	34.64	8.79	34.99	8.88
	0.23	0.08	0.22	0.06
sta83 IIc	33.74	3.4	34.26	3.39
uta92	34.65	3.43	34.99	3.43
	0.25	0.02	0.22	0.02
sta83 IIc	34.28	3.38	34.26	3.35
uta92 II	34.66	3.42	34.99	3.41
	0.19	0.01	0.22	0.03
sta83 IIc	34.08	27.32	34.26	28.31
ute92	34.64	28.2	34.99	28.75
	0.27	0.35	0.22	0.14
sta83 IIc	34.21	40.72	34.26	41.05
yor83	34.69	41.95	34.99	42.87
	0.22	0.57	0.22	2.91
tre92	8.6	3.38	8.78	3.39
uta92	8.8	3.43	8.88	3.43
	0.07	0.02	0.06	0.02
tre92	8.58	3.39	8.78	3.35
uta92 II	8.81	3.41	8.88	3.41
	0.1	0.02	0.06	0.03
tre92	8.66	27.23	8.78	28.31
ute92	8.8	28.24	8.88	28.75
	0.09	0.36	0.06	0.14
tre92	8.67	40.66	8.78	41.05
yor83	8.8	41.69	8.88	42.87
	0.07	0.48	0.06	2.91
uta92	3.38	3.37	3.39	3.35
uta92 II	3.43	3.41	3.43	3.41
	0.02	0.02	0.02	0.03
uta92	3.39	27.88	3.39	28.31
ute92	3.43	28.26	3.43	28.75
	0.02	0.24	0.02	0.14
uta92	3.39	39.59	3.39	41.05
yor83	3.43	42.13	3.43	42.87
	0.02	2.22	0.02	2.91
uta92 II	3.37	27.5	3.35	28.31
ute92	3.42	28.28	3.41	28.75
	0.01	0.31	0.03	0.14
uta92 II	3.37	40.62	3.35	41.05
yor83	3.41	41.7	3.41	42.87
	0.02	0.41	0.03	2.91
ute92	27.2	40.45	28.31	41.05
yor83	28.27	41.95	28.75	42.87
	0.36	0.7	0.14	2.91

significantly better than those obtained by approach *B*, the count of all-wins for *A* is increased by one.

2) *One-Win:* If approach *A* achieves a significantly better result on one of the two tasks, and better, worse or equal results than *B* on another task, then the count of one-wins for *A* is increased by one.

3) *Tie:* Otherwise, increase the tie count by one.

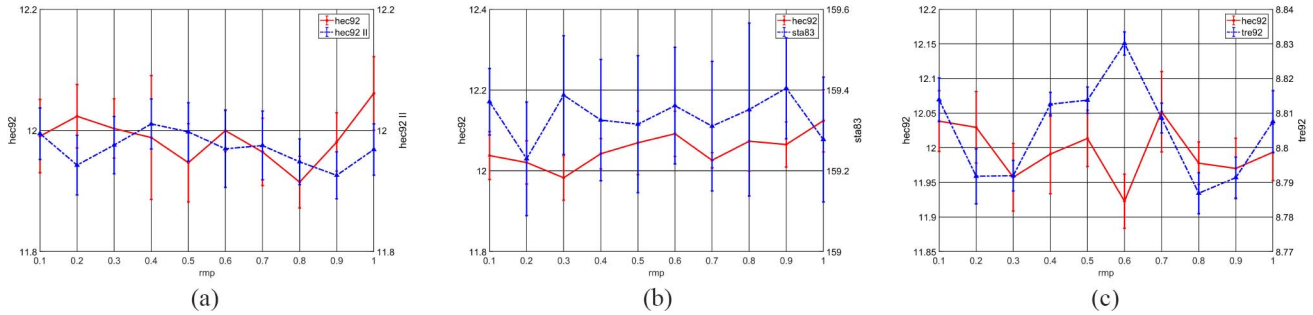

Fig. 4. Impact of different probabilities *rmp* on EMHH. (a) (hec92, hec92 II). (b) (hec92, sta83). (c) (hec92, tre92).

TABLE II
STATISTIC COMPARISONS BETWEEN EMHH AND SOHH ON RESULTS IN TABLE I

All-win		One-win		Tie
EMHH	SOHH	EMHH	SOHH	13
43	0	63	1	

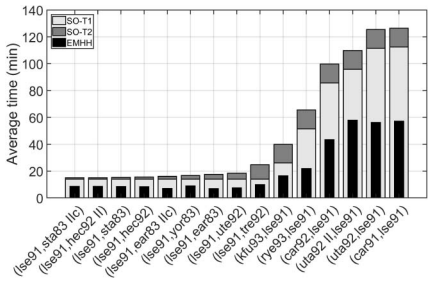

Fig. 5. Time comparison between SOHH and EMHH for MFO problems.

The comparison results show that EMHH is highly effective against SOHH on uETTPs. It outperformed SOHH on 43 MFO problems, achieved significantly better results for one of the two tasks than SOHH on 63 MFO problems, and performed similarly on 13 MFO problems. SOHH won on only one MFO problem in terms of the one-win indicator. Moreover, the results demonstrate that multitasking works effectively in the low-level heuristic space. In summary, EMHH is shown to be an effective hyper-heuristic framework for solving uETTPs.

2) Comparisons Between the Computational Costs of EMHH and SOHH: Compared with SOHH under the same parameter settings, the number of evaluations in EMHH can be reduced to almost $1/K$ of that in SOHH. In EMHH, each individual is evaluated on only one of the K tasks indicated by its skill factor, which eventually leads to a reduction in computational costs. As the baseline, Table III presents the average time used by SOHH to solve the 16 uETTP instances. To demonstrate the efficiency of EMHH, the average computational costs for solving all MFO problems comprising instance lse91 and the other 15 instances are presented in Fig. 5. For comparison, SOHH is used to solve both single tasks sequentially in these MFO problems (i.e., the average time consumed by the first and second tasks are denoted as SO-T1 and SO-T2, respectively, in Fig. 5). Instance lse91 consumes the median average time according to Table III; thus, the performance of EMHH on multitasking lse91 with the other instances consuming longer and shorter time can be clearly presented.

In most of the cases shown in Fig. 5, the time consumed by EMHH is approximately half of that by SOHH. Moreover, EMHH significantly reduces the time consumed by the first task that represents the computationally expensive task in an MFO. Note that the orders of tasks in an MFO do not affect the performance of EMHH; here we subject the computationally expensive task to T_1 for demonstration purposes. We can conclude that EMHH works more efficiently in the low-level heuristic space than SOHH in solving uETTPs. The same conclusion can be drawn from the time comparisons of other instances, as shown in Fig. S1 in the supplementary material.

3) Generality on Asynchronous Optimization: The results in Table II indicate that in the low-level heuristic space, the solutions for different instances may share some commonly evolved knowledge. Although the direct timetable solutions for different instances could be highly distinctive, the low-level heuristic sequences used to construct them may possess similar patterns, which subsequently indicates that the heuristic sequences obtained in solving one task could be reused for solving other tasks. To verify this, another set of experiments is conducted, where a single task is first optimized by SOHH; then, after a number of iterations, the second task is inserted into the evolution; thus, the paradigm turns into EMHH for the remaining generations.

Based on the average time shown in Table III, instance hec92 consumes relatively less time than other instances, and, therefore, is processed first as task T_1 to save computational resources. All the parameters are kept unchanged and the second task is started after half of the total generations.

As seen from Table IV, compared with SOHH, EMHH achieved significantly better results on T_2 in ten test cases, better results in three, and equal result in one test case; while SOHH obtained a better result in only one test case. Although the second task is introduced halfway through the evolution, EMHH still outperformed SOHH. This demonstrates that the low-level heuristic sequences learned from optimizing hec92 are applicable to other instances, i.e., the solutions of different uETTP instances in the low-level heuristic space may share some common patterns or knowledge. Moreover, EMHH is capable of retaining such knowledge in the low-level heuristic space via the knowledge-transfer scheme, showing a higher level of generality on asynchronous optimization.

TABLE III
AVERAGE TIME CONSUMED BY SOHH IN SOLVING CARTER BENCHMARKS (IN MINUTES)

Instance	sta83 IIc	hec92 II	sta83	hec92	ear83 IIc	yor83	ear83	ute92
Av. Time	1.03	1.09	1.36	1.62	2.04	2.64	3.48	4.42
No. of exams	138	80	139	81	189	181	190	184
Instance	tre92	lse91	kfu93	rye93	car92	uta92 II	uta92	car91
Av. time	10.77	13.99	26.08	51.52	85.69	95.86	111.49	112.46
No. of exams	261	381	461	486	543	638	622	682

TABLE IV
COMPARISONS BETWEEN EMHH AND SOHH, WHERE IN EMHH THE SECOND TASK IS TRIGGERED IN LATE GENERATIONS

MFOs	EMHH	SOHH	p-value	MFOs	EMHH	SOHH	p-value	MFOs	EMHH	SOHH	p-value
	T_1	T_2			T_1	T_2			T_1	T_2	
hec92	5.19	5.18		hec92	15.05	15.37		hec92	8.55	8.78	
car91	5.22	5.26	1.49E-04	kfu93	15.54	15.68	2.38E-03	tre92	8.79	8.88	2.58E-06
	0.03	0.04			0.17	0.15			0.08	0.06	
hec92	4.33	4.31		hec92	11.29	11.65		hec92	3.38	3.39	
car92	4.39	4.41	8.13E-03	lse91	11.61	11.85	5.46E-06	uta92	3.42	3.43	5.35E-01
	0.03	0.04			0.16	0.16			0.02	0.02	
hec92	35.88	36.62		hec92	9.36	9.34		hec92	3.37	3.35	
ear83	36.75	37.12	2.21E-03	rye93	9.59	9.55	6.57E-02	uta92 II	3.41	3.41	8.48E-01
	0.54	0.25			0.1	0.08			0.02	0.03	
hec92	39.58	39.51		hec92	158.61	169.77		hec92	27.9	28.31	
ear83 IIc	40.71	41.13	9.19E-02	sta83	159.35	169.86	2.95E-11	ute92	28.4	28.75	5.53E-08
	0.7	1			0.46	0.09			0.24	0.14	
hec92	11.51	11.67		hec92	34.28	34.26		hec92	40.21	41.05	
hec92 II	11.95	12.1	7.98E-02	sta83 IIc	34.72	34.99	7.73E-06	yor83	41.74	42.87	7.29E-03
	0.19	0.29			0.2	0.22			0.55	2.91	

4) Generality on Synchronous Cross-Domain Optimization: To examine the generality of EMHH on synchronous cross-domain optimization, 16 MFO problems are created, each consisting of a uETTP task and its GCP variant. All the parameters used in this section are consistent with those in the above experiments. The comparison between EMHH and SOHH on solving these cross-domain MFO problems and their single-objective optimization variants, respectively, are presented in Table V.

As seen from Table V, for uETTP tasks (T_1), EMHH obtained significantly better and better results than SOHH in nine and five instances, respectively, and equal results in the other two instances. For all the graph-coloring tasks (T_2), both approaches obtained the same best results except for instance yor83, where the performance of EMHH is slightly more robust than that of SOHH. To conclude, the results in Table V demonstrate a high generality of EMHH on synchronous cross-domain optimization. Moreover, in the low-level heuristic space, such as the graph heuristic space here, the solutions for different problems, such as uETTPs and GCPs, may share some common patterns evolved by solving the companion tasks in MFO. Therefore, the knowledge in the solutions of the heuristic space obtained from optimizing in one domain could be utilized by other domains, which could reduce the computational cost significantly and, more importantly, may accelerate the convergence of optimization in every domain.

5) Generality on Many-Task Optimization Problems: To further evaluate the generality of EMHH, experiments on many-task optimization (MTO) problems that include more than two tasks are conducted. The problem sets include: 1) six three-task problems and two five-task problems, with all uETTP tasks and 2) two four-task cross-domain problems with two uETTP tasks and two GCP tasks. Tables SIV, SV, and SVI in the supplementary material present the three-task, four-task, and five-task problem sets, respectively. These problem sets are designed to cover MTO problems with tasks of various scales, i.e., dimensions. For example, the scales of the tasks in Set 1 are close to each other, while the middle-scale task tre92 in Set 2 and the large-scale task car91 in Set 3 are of

TABLE V
MINIMUM, AVERAGE, AND STANDARD DEVIATIONS FOR CROSS-DOMAIN MFO PROBLEMS

MFOs	EMHH		SOHH		MFOs	EMHH		SOHH	
	T_1	T_2	T_1	T_2		T_1	T_2	T_1	T_2
car91	5.17	30	5.18	30	rye93	9.45	21	9.34	21
car91	5.23	31	5.26	31	rye93	9.59	22	9.55	22
	0.03		0.04			0.08		0.08	
car92	4.32	29	4.31	29	sta83	158.54	13	169.77	13
car92	4.38	30	4.41	30	sta83	159.35	13	169.86	13
	0.03		0.04			0.35		0.09	
ear83	35.96	22	36.62	22	sta83 IIc	34.16	35	34.26	35
ear83	36.91	23	37.12	23	sta83 IIc	34.64	35	34.99	35
	0.71		0.25			0.22		0.22	
ear83 IIc	39.02	23	39.51	23	tre92	8.61	20	8.78	20
ear83 IIc	40.19	24	41.13	24	tre93	8.81	20	8.88	20
	0.61		1			0.07		0.06	
hec92	11.52	17	11.69	17	uta92	3.4	31	3.39	31
hec92	12.05	18	12.12	18	uta92	3.43	31	3.43	31
	0.26		0.27			0.02		0.02	
hec92 II	11.43	17	11.67	17	uta92 II	3.36	30	3.348	30
hec92 II	11.93	18	12.1	18	uta92 II	3.41	31	3.41	31
	0.23		0.29			0.02		0.03	
kfu93	15.15	19	15.37	19	ute92	27.59	10	28.31	10
kfu93	15.58	19	15.68	19	ute92	28.17	10	28.75	10
	0.16		0.15			0.29		0.14	
lse91	11.26	17	11.65	17	yor83	40.7	20	41.05	20
lse91	11.67	17	11.85	17	yor83	42.04	20	42.87	21
	0.21		0.155			0.67		2.91	

TABLE VI
COMPARISONS BETWEEN EMHH AND A SELECTION OF ALGORITHMS ON UETTPS

Instance	Asm04	Asm07	Asm09	Bur07	Pil07	Pil09a	Pil09b	Qu09a	Qu09b	Sab12	Qu15	EMHH		
						f_{min}						$f_{i-min-min}$	$f_{i-avg-min}$	f_{c-min}
car91	5.29	5.19	5.29	5.36	–	4.97	–	5.3	5.11	5.14	4.95	5.11	5.15	5.17
car92	4.56	4.32	4.54	4.53	–	4.28	–	4.7	4.32	4.7	4.09	4.27	4.32	4.32
ear83	37.02	36.16	37.02	37.92	36.74	36.86	37.39	35.54	35.56	37.86	34.97	35.31	35.75	35.96
ear83 IIc	–	–	–	–	–	–	–	–	39.38	–	–	38.94	39.31	39.02
hec92	11.78	11.6	11.78	12.25	11.55	11.85	11.43	11.78	11.62	11.9	11.11	11.45	11.59	11.51
hec92 II	–	–	–	–	–	–	–	–	11.5	–	–	11.39	11.57	11.43
kfu93	15.81	15.03	15.8	15.2	14.22	14.62	–	15.09	15.18	15.3	14.09	14.79	15.14	15.15
lse91	12.09	11.35	12.09	11.33	10.9	11.14	–	12.71	11.32	12.33	10.71	11.07	11.26	11.26
rye93	10.35	9.75	10.38	–	9.35	9.65	–	–	–	10.71	9.2	9.3	9.39	9.45
sta83	160.42	158.64	160.42	158.19	158.22	158.33	158.38	159.2	158.88	160.12	157.64	158.5	158.62	158.54
sta83 IIc	–	–	–	–	–	–	–	–	34.32	–	–	33.74	34.16	34.15
tre92	8.67	8.47	8.67	8.75	8.48	8.48	–	8.67	8.52	8.32	8.27	8.55	8.62	8.61
uta92	3.57	3.52	3.57	3.88	–	3.4	–	3.32	3.21	3.88	3.33	3.38	3.39	3.4
uta92 II	–	–	–	–	–	–	–	–	3.45	–	–	3.34	3.37	3.36
ute92	27.28	27.55	28.07	28.01	26.65	28.88	27.31	30	28	32.67	26.18	27.2	27.52	27.59
yor83	40.66	39.25	39.8	41.37	41.57	40.74	39.96	40.24	40.71	40.53	37.88	39.59	40.54	40.7
INS (12)	9.5	6.3	10.2	10.5	5.1	6.5	5.8	9	6.6	10.3	1.2	3.6	6.3	6.9
INS (16)	–	–	–	–	–	–	–	–	3.8	–	–	1	3.3	2.3

"–" indicates that the corresponding instances are not tested or the calculation of that value is nonsense. The solutions that are compared against our approach on uETTPs include:
Asm04: Fuzzy combinations of two ordering criteria [41].
Asm07: Fuzzy combinations of multiple ordering criteria [42].
Asm09: Asm04 with turning [43].
Bur07: Tabu search [33].
Pil07: A genetic programming-based hyper-heuristic [44].
Pil09a: Four hierarchical combination of heuristics [45].
Pil09b: A genetic programming hyper-heuristic to evolve functions of low-level heuristic and logical operators [46].
Qu09a: Four local search [47].
Qu09b: An adaptive hybridization of heuristics [34].
Sab12: A graph coloring constructive hyper-heuristics where the hybridizations of four heuristic sequence are utilized to order exams [48].
Qu15: EDA-based hyper-heuristic [5].

even larger differences among the tasks in these two problem sets. We keep all parameters consistent with the above experiments. The experimental results of 30 independent runs on Sets 1–6, Sets 7–8, and Sets 9–10 are shown in Tables SVII, SVIII, and SIX, respectively, in the supplementary material.

As seen from Table SVII in the supplementary material, the proposed EMHH outperformed SOHH in all three-task problem sets, except that in Set 5, SOHH obtained better average objective value in T_1. The results in Table SIX in the supplementary material also demonstrate that EMHH can still perform better than SOHH on five-task problems. Moreover, in Table SVIII in the supplementary material, EMHH performs equally to SOHH on GCP tasks but slightly better than SOHH on uETTP tasks. Given that the total generations used in EMHH and SOHH are identical, the average computational resources allocated to each task in three, four, and five-task problems in EMHH are only one-third, a quarter, and one-fifth, respectively, of that used in SOHH. Based on these observations, we can conclude that the proposed EMHH still generalize well on MTO problems with more.

6) Comparisons of EMHH With Other Hyper-Heuristics: The above comparisons are between the conventional EA-based and the multitask EA-based hyper-heuristics. In this section, we compare our approach with other state-of-the-art hyper-heuristics in the literature on both uETTPs and GCPs. The chosen algorithms for comparison include the selection constructive hyper-heuristics with graph heuristics and were evaluated on uETTPs and GCPs in Tables VI and VII, respectively. Note that the purpose of this comparison is to provide an overall performance evaluation on the proposed method from another point of view. There might be other indicators or criteria that may provide comparisons on different aspects of the algorithms.

Table VI presents three indicators in the last three columns. $f_{\mathrm{i-min-min}}$ refers to the best of the minimum objective value obtained by EMHH in solving the intradomain MFO problems that consist of the pure uETTP tasks shown in Table I for each instance. $f_{\mathrm{i-avg-min}}$ represents the average of the minimum objective value of each instance in solving these MFO problems. The minimum objective value produced by EMHH in solving the cross-domain MFO problems, i.e., problems that comprise uETTP and GCP tasks, are denoted by $f_{\mathrm{c-min}}$. In the last two rows of Table VI, the average ranks over all instances (INS (16)) and instances excluding ear83 IIc, hec92 II, sta83 IIc, and uta92 II (INS (12)) from all the compared algorithms based on their best performance are provided. Due to the lack of competitive algorithms in the existing literature, the INS(16) ranks in Table VI are provided for future comparisons. The details of the ranks for each compared algorithm on each instance can be found in Table SII in the supplementary material.

From Table VI, we can see that EMHH ranks the second and sixth out of 14 competitors using $f_{\mathrm{i-min-min}}$ and $f_{\mathrm{i-avg-min}}$ regarding the INS (12) ranks, respectively. This indicates that EMHH is competitive on the intradomain MFO problems. Particularly, given that EMHH optimizes multiple instances in a single run, the results strongly demonstrate the high generality of EMHH. Additionally, EMHH ranks the ninth using $f_{\mathrm{c-min}}$ in Table VI, and in Table VII, it obtains competitive results on GCPs as well. Thus, the proposed EMHH has better generality than the existing hyper-heuristics, given the

TABLE VII
MINIMUM NUMBER OF COLORS FOUND BY EMHH AND THE SELECTED ALGORITHMS

	car91	car92	ear83	ear83 IIc	hec92	hec92 II	kfu93	lse91
Qu09	30	29	22	–	17	–	19	17
Qu15	**28**	**27**	22	–	17	–	19	17
EMHH	30	29	22	23	17	17	19	17
	rye93	sta83	sta83 IIc	tre92	uta92	uta92 II	ute92	yor83
Qu09	–	13	–	20	31	–	10	19
Qu15	21	13	–	20	**29**	–	10	18
EMHH	21	13	35	20	31	30	10	20

"–" indicates that instances are not tested. Bold values represent the best solutions among all competitors, and the optimal results are underlined. The solutions that are compared against our approach on GCPs include:
Qu09b: An adaptive hybridization of heuristics [34].
Qu15: EDA-based hyper-heuristic [5].

competitive results and its ability to tackle instances from different problem domains simultaneously, which has not been addressed by other hyper-heuristics in the literature.

E. Preliminary Discussions

To analyze success of EMHH, the landscape of the heuristics search space consisting of sequences of graph heuristics for the tested problems should be analyzed first. In previous work [32], the trends of hybridizing LWD with SD in the obtained best heuristic sequences for both uEPPTs and GCPs were statistically analyzed. The visualized results indicated that the best heuristic sequences vary significantly among different instances, whether from the same or different problem domain. However, the overall trend is that they employ more LWD in the early stages of the solution construction than in the later stages. This presents a common pattern of different problems in the heuristic space. Thus, one of our conjectures for the success of EMHH is that the knowledge-transfer mechanism can promote the exchange of the learned patterns among tasks, which eventually facilitates the convergence of all tasks.

Furthermore, according to [36], the landscape of the heuristics search space for uETTPs has the following features: *big valley structure*, *large number of local optima*, *high ruggedness*, *wide plateaus*, *shallow valleys*, and *positional bias*. Based on these features, it can be inferred that search methodologies that work in the search space under discussion may easily get stuck in local optima, especially considering that there are wide plateaus in the search space. Therefore, we suspect that part of the superiority of EMHH is that it can promote the diversity of the heuristic sequences via the knowledge-transfer mechanism, thus improving the possibility of jumping out of the local optima. To verify this conjecture, we observed the best ten heuristic sequences for all the instances based on the results of EMHH and SOHH from Table I. The MFO problem (ear83, sta83) is selected as the representative due to the significant distinction between the best ten heuristic sequences shown as Fig. S2 in the supplementary material. In Fig. S2(a) and (b) in the supplementary material, the left 3 heuristics in the selected heuristic sequences of ear83 and sta83 are obviously different. Therefore, EMHH intuitively would suffer from a negative transfer. However, Table I shows that EMHH achieved significantly better results than SOHH in both tasks. Furthermore, from Fig. S2 in the supplementary material, we can see that the results obtained by EMHH are clearly closer to the best ten heuristic sequences than those obtained by SOHH. Namely, SOHH is stuck in local optima, while EMHH successfully escaped due to the knowledge-transfer mechanism. Note that in [23], a same conjecture has been made in the permutation-based encoding search space, but verification was not provided. Finally, there may be negative transfers [18] in the process of knowledge-transfer, as in the cases of MFO problems that including rye93, where SOHH achieved better results than EMHH on rye93.

F. Potential Applications and Future Research Challenges

Other educational timetabling problems are possible applications that require multitasking hyper-heuristics. Another scenario is the optimization of cloud services, where the server is likely to face concurrent service requests from multiple customers. The optimization of these requests might scale differently or even belong to different domains. In this regard, the multitasking hyper-heuristics present an appropriate approach with a reduced computational cost from the multitasking and knowledge-transfer schemes. In addition, the applications of hyper-heuristics as presented in [10], such as vehicle routing problems, nurse rostering problems and packing problems, might also be potential applications.

As mentioned in the Introduction, hyper-heuristics can be classified into four categories, namely, selection constructive/perturbative, generation constructive/perturbative. This article concerns the selection constructive hyper-heuristic as the precursor of extending the hyper-heuristics into the multitasking scheme. Extensions of other classes of hyper-heuristics remain to be studied. Several challenges may be encountered in these extensions. The first is the design of the unified representation. Although low-level heuristics constitute the search space of hyper-heuristics, they are domain-specific; that is, the graph heuristics used to solve educational timetabling problems might not be suitable to solve other combinational problems. Thus, careful attention should be paid to the design of the unified representation when handling problems with very distinct heuristics. Second, much effort is needed to develop multitasking hyper-heuristics that combine different categories of hyper-heuristics; for example, selection and generation constructive hyper-heuristics. Third, the design of efficient knowledge-transfer schemes in the heuristic space challenging as well.

V. CONCLUSION

In this article, a unified framework of EMHH is proposed, where the concept of evolutionary multitasking and graph heuristics are used as the high-level search methodology and low-level heuristics, respectively. The EMHH has been evaluated on examination timetabling and graph coloring problems. Given that the purpose is to propose a new general framework instead of competing with particular algorithms on specific single problems, we found the results encouraging.

In conclusion, the superiorities of EMHH compared with the conventional single-tasking hyper-heuristic are twofold.

1) It raises the generality of both the hyper-heuristic and evolutionary multitasking to a higher level; i.e., it extends the generality of hyper-heuristics in addressing multiple optimization problems and the scope of unification in evolutionary multitasking.
2) EMHH is more effective and efficient. To be specific, on the one hand, it can take advantages of the commonalities among tasks to facilitate the convergence of the algorithm. On the other hand, the search biases provided by different tasks via the knowledge-transfer mechanism can promote the diversity in the heuristic space, thus improving the global search of EMHH.

In the future, we will investigate EMHH on other problem domains. The properties of common solution structures in a high-level space need to be further examined. The design of more effective mechanisms to adapt the reusable knowledge across multiple domains could be another interesting future research direction.

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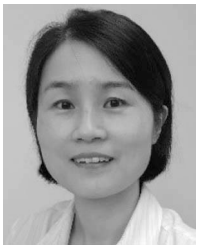

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