

The Network Operator Method for Synthesis of Intelligent Control System

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Abstract—Application of the network operator for the synthesis of intelligent control systems is considered. An example of the synthesis of automatic control on the space trajectories of unmanned helicopter is given.

Synthesis of control systems, intelligent systems, network operator, quadrotor.

I. INTRODUCTION

Intelligent control systems are used at the control of complex objects for decision making and choice of operation options. A distinctive feature of intelligent control is the system with logic conclusions. The mathematical model of intelligent control systems is described by mathematical and logical expressions.

In this paper we consider a numerical method for the synthesis of intelligent control systems. For the synthesis problem the control is searched in a form of multidimensional function depending from coordinates of space of object conditions. A solution of the synthesis is the mathematical expression for the synthesis control function.

Creation of numerical method for search the mathematical expressions became possible when the genetic programming appeared [1, 2]. The method of genetic programming allows to search mathematical expressions in a form of a symbol string by using a genetic algorithm.

The network operator method uses a directed graph to describe the mathematical expression [3]. The network operator method has been successfully used to solve problems of synthesis of control systems [4, 5, 6, 7, 8].

This paper presents the use of the network operator for the synthesis of intelligent control systems. The developed method

allows to simultaneously find a mathematical expression for synthesizing control functions and the Boolean expression for the function of logic outputs.

II. THE PROBLEM OF SYNTHESIS OF INTELLIGENT CONTROL SYSTEM

The system of differential equations which describes the dynamics of the object is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}), \quad (1)$$

where $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in U \subseteq \mathbf{R}^m$, U is a closed bounded domain.

The initial conditions are

$$\mathbf{x}(0) = \mathbf{x}^0. \quad (2)$$

The quality criterion is given in the form

$$J = \int_0^{t_f} f_0(\mathbf{x}(t), \mathbf{u}(t)) dt \rightarrow \text{minimum}, \quad (3)$$

where t_f is a control time.

To minimize functional (3) it is necessary to find the control in a form

$$\mathbf{u} = \mathbf{g}(\mathbf{x}, \mathbf{v}), \quad (4)$$

where \mathbf{v} is a vector of Boolean variables, $\mathbf{v} = [v_1 \dots v_k]^T$, $v_i \in \{0,1\}$, $i = \overline{1, k}$.

Boolean variables define the choice of control options based predicate functions, which is also necessary to find

$$\mathbf{v} = \mathbf{h}(\mathbf{x}), \quad (5)$$

where $\mathbf{h}(\mathbf{x}): \mathbf{R}^n \rightarrow \overbrace{\{0,1\} \times \dots \times \{0,1\}}^k$.

III. THE NETWORK OPERATOR METHOD

Consider the structure of mathematical expression. Many mathematical expressions are described by the following types of elements: operations, parameters and variables. There are unary and binary operations.

Thus, we have constructive sets to describe mathematical expressions. Let us fix symbols to correspond to the elements of these sets.

Set of variables

$$X = (x_1, \dots, x_n). \quad (6)$$

Set of parameters

$$Q = (q_1, \dots, q_P). \quad (7)$$

Unary operations set

$$O_1 = (\rho_1(z) = z, \rho_2(z), \dots, \rho_W(z)). \quad (8)$$

Binary operations set

$$O_2 = (\chi_0(z', z''), \dots, \chi_{V-1}(z', z'')). \quad (9)$$

The set of unary operations must include the identity operation $\rho_1(z) = z$. Binary operations are commutative $\chi_i(z', z'') = \chi_i(z'', z')$, $i = \overline{0, V-1}$, associative $\chi_i(z', \chi_i(z'', z''')) = \chi_i(\chi_i(z', z''), z''')$, $i = \overline{0, V-1}$, and have a unit element $\forall \chi_i(z', z'') \in O_2 \exists e_i$ such that $\chi_i(e_i, z) = z$, $i = \overline{0, V-1}$.

When a mathematical expression is represented by the elements of the constructive sets (6) – (9), it is called the program notation of the mathematical expression.

To form a graph associated with the mathematical expression, a program notation has to satisfy additional requirements. Only unary operations or 1 are used as arguments of binary operations. Binary operations or an element from the sets of variables and parameters are used as arguments for unary operations. Those unary operations that have the same parameters or variables as arguments are not arguments for binary operations.

All the requirements mentioned above are satisfied if we use in the program notation additional unary identity operation and unit elements for binary operations.

The main principle for constructing a graph based on the graphical notation of mathematical expressions is the following: an arc of the graph corresponds with the unary operation, and a node corresponds with the binary operation, parameter or variable.

For construction a network operator we can use the following unary and binary operations:

$$\rho_1(z) = z, \rho_2(z) = \begin{cases} \varepsilon^{-1}, & \text{if } |z| > \frac{1}{\sqrt{\varepsilon}} \\ z^2, & \text{otherwise} \end{cases}, \rho_3(z) = -z,$$

$$\rho_4(z) = \text{sgn}(z)\sqrt{|z|}, \rho_5(z) = \begin{cases} \frac{\text{sgn}(z)}{\varepsilon}, & \text{if } |z| < \varepsilon \\ \frac{1}{z}, & \text{otherwise} \end{cases},$$

$$\rho_6(z) = \begin{cases} \varepsilon^{-1}, & \text{if } z > -\ln(\varepsilon) \\ e^z, & \text{otherwise} \end{cases}, \rho_7(z) = \begin{cases} \ln(\varepsilon), & \text{if } -\ln|z| > \frac{1}{\varepsilon} \\ \ln|z|, & \text{otherwise} \end{cases},$$

$$\rho_8(z) = \begin{cases} \text{sgn}(z), & \text{if } |z| > -\ln(\varepsilon) \\ \frac{1-e^{-z}}{1+e^{-z}}, & \text{otherwise} \end{cases}, \rho_9(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{otherwise} \end{cases},$$

$$\rho_{10}(z) = \text{sgn}(z), \rho_{11}(z) = \cos(z), \rho_{12}(z) = \sin(z),$$

$$\rho_{13}(z) = \arctan(z), \rho_{14}(z) = \begin{cases} \frac{\text{sgn}(z)}{\varepsilon}, & \text{if } |z| > \frac{1}{\sqrt[3]{\varepsilon}} \\ z^3, & \text{otherwise} \end{cases},$$

$$\rho_{15}(z) = \begin{cases} \sqrt[3]{\varepsilon}, & \text{if } |z| < \varepsilon \\ \sqrt[3]{z}, & \text{otherwise} \end{cases}, \rho_{16}(z) = \begin{cases} z, & \text{if } |z| < 1 \\ \text{sgn}(z), & \text{otherwise} \end{cases},$$

$$\rho_{17}(z) = \text{sgn}(z)\ln(|z|+1),$$

$$\rho_{18}(z) = \begin{cases} \frac{\text{sgn}(z)}{\varepsilon}, & \text{if } |z| > -\ln(\varepsilon) \\ \text{sgn}(z)(e^{|z|}-1), & \text{otherwise} \end{cases},$$

$$\rho_{19}(z) = \begin{cases} 0, & \text{if } |z| > -\ln(\varepsilon) \\ \text{sgn}(z)e^{-|z|}, & \text{otherwise} \end{cases},$$

$$\rho_{20}(z) = \begin{cases} 1, & \text{if } z > \varepsilon \\ 0, & \text{if } z < 0 \\ \frac{3z^2}{\varepsilon^2} - \frac{2z^3}{\varepsilon^3}, & \text{otherwise} \end{cases},$$

$$\rho_{21}(z) = \begin{cases} 1, & \text{if } z > \frac{\varepsilon}{2} \\ -1, & \text{if } z < -\frac{\varepsilon}{2} \\ \frac{3z}{\varepsilon^2} - \frac{4z^3}{\varepsilon^3}, & \text{otherwise} \end{cases},$$

$$\rho_{22}(z) = \begin{cases} 0, & \text{if } |z| > -\ln(\varepsilon) \\ e^{-|z|}, & \text{otherwise} \end{cases},$$

$$v = y_1 \wedge \neg y_2 \vee y_3 \wedge y_4.$$

We use operations from sets (10), (11) and obtain

$$v = \omega_0(\omega_1(y_1, \varphi_2(y_2)), \omega_1(y_3, y_4)).$$

To design graph for this logical expression we should transform it so that binary operations had like arguments unary operations, and arguments of unary operations were binary operation or variables. For this goal we add to the logical expression identity unary operation and binary operations with unit element. We obtain

$$v = \omega_0(\varphi_1(\omega_1(\varphi_1(y_1), \varphi_2(y_2))), \omega_1(\varphi_1(y_3), \varphi_1(y_4))).$$

Graph of logical network operator is represented in Fig. 2.

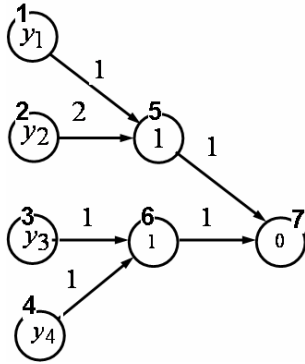


Figure 2. Example of a logical network operator

The matrix of the logical network operator is

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

VI. GENETIC ALGORITHM FOR SYNTHESIS OF INTELLIGENT CONTROL SYSTEM

To construct the algorithm for synthesis of optimal intelligent control system we use a principle of basic solution. The principle of basic solution means that when solving optimization problems, initially we set the basic solution that is one of admissible solutions, then define small variations of basic solution and create search algorithm that searches for the optimal solution on the set of small variations.

Let us define the following small variations of network operator: 0 - replacement of unary operation by the edge of the graph; 1 - replacement of binary operation in the node; 2 - addition of the edge with unary operation; 3 - removal of unary operation with the edge of the graph. These all variations can be used for a logical network operator too.

To describe any variation it is enough to use an integer vector $\mathbf{w} = [w_1 \ w_2 \ w_3 \ w_4]^T$ of four elements. The first

element of vector of variations \mathbf{W} specifies the number of variation, the second – the number of node that the edge comes out, the third - the number of node that the edge comes in, and the fourth – number of unary or binary operation.

For genetic algorithm we should create a basic solution that is described by the basic network operators Ψ^0 and Ω^0 . The ordered set of vectors of variations is considered to be a chromosome

$$\mathbf{W}^i = (\mathbf{w}^{i1}, \dots, \mathbf{w}^{il}), \quad i = \overline{1, H}, \quad (12)$$

where H is the size of population, l is the length of chromosome.

To know the network operator on which the vector of variations acts, we introduce a binary vector

$$\mathbf{s}^i = [s_1^i \ \dots \ s_l^i]^T, \quad s_j^i \in \{0, 1\}, \quad i = \overline{1, H}, \quad j = \overline{1, l}. \quad (13)$$

All genetic operations are performed on the vectors of variations. Each vector of variations \mathbf{W}^i changes basic solution thus we obtain new solution

$$\Psi^{i,j} = \begin{cases} \Psi^{i,j-1}, & \text{if } s_j^i = 1, \\ \mathbf{w}^{ij} \circ \Psi^{i,j-1} - \text{otherwise,} & j = \overline{1, l}, \end{cases} \quad (14)$$

$$\Omega^{i,j} = \begin{cases} \Omega^{i,j-1}, & \text{if } s_j^i = 0, \\ \mathbf{w}^{ij} \circ \Omega^{i,j-1} - \text{otherwise,} & j = \overline{1, l}. \end{cases} \quad (15)$$

For effective search we change basic solutions on the best solutions found after some number of generations called epoch.

During search of mathematical expression structure which is described with network operators Ψ^i and Ω^i , we use genetic algorithm and for search of optimal values of parameters $\mathbf{q} \in Q$.

Thus in genetic algorithm each chromosome consists of two parts, a structural part \mathbf{W}^i with a binary vector \mathbf{s}^i and parametrical one \mathbf{q}^i .

After crossover of two chromosomes we receive four children. For two children only parametrical parts are crossed, and structural parts are not changed. For two other children both structural and parametrical parts are changed.

VII. AN EXAMPLE

Consider the synthesis of intelligent control system for unmanned helicopter named quadrotor.

The mathematical model of quadrotor is the following

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\frac{T}{m_0} \cos(x_7) \sin(x_9),$$

$$\begin{aligned}\dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{T}{m_0} \cos(x_7) \cos(x_9) - g_0,\end{aligned}$$

$$\begin{aligned}\dot{x}_5 &= x_6, \\ \dot{x}_6 &= \frac{T}{m_0} \sin(x_6),\end{aligned}$$

$$\begin{aligned}\dot{x}_7 &= x_8, \\ \dot{x}_8 &= \frac{(u_1 - u_3)l}{I_1},\end{aligned}$$

$$\begin{aligned}\dot{x}_9 &= x_{10}, \\ \dot{x}_{10} &= \frac{(u_2 - u_4)l}{I_3},\end{aligned}$$

where x_1, x_3, x_5 are the coordinates of centre of mass, x_1 is a linear range, x_3 is an altitude, x_5 is a cross range, x_2, x_4, x_6 are corresponding projections of velocity vector of center of mass, x_7, x_9 are rotational angles over horizontal plane, x_8, x_{10} are corresponding velocity angles, u_1, u_2, u_3, u_4 are thrusts of propellers, $T = u_1 + u_2 + u_3 + u_4$, l is the distance between the opposite propellers, I_1, I_2 are moments of inertia toward the horizontal, m_0 is a quadrotor mass, g_0 is a gravity acceleration.

The control is bounded by

$$u^- \leq u_i \leq u^+, \quad i = \overline{1,4},$$

where u^- and u^+ are given values of minimum and maximum thrust of propeller.

The trusts of propellers should satisfy the following condition

$$u_1 - u_2 + u_3 - u_4 = 0,$$

Rotational angles x_7 and x_9 are bounded by

$$x_7^- \leq x_7 \leq x_7^+,$$

$$x_9^- \leq x_9 \leq x_9^+.$$

Space trajectory is presented as a set of points

$$P = \left((x_1^0, x_3^0, x_5^0), \dots, (x_1^{M-1}, x_3^{M-1}, x_5^{M-1}) \right),$$

where M is a number of points on the trajectory.

It is necessary to find the control that minimizes two object functions. The first function determines accuracy of movement over the trajectory. The second function defines time of passing of a trajectory.

$$\begin{aligned}J_1 &= \sum_{j=1}^{M-1} \min_t \left\{ \sqrt{\sum_{\alpha} (x_{\alpha}(t) - x_{\alpha}^j)^2} \right\} \rightarrow \min, \\ J_2 &= t_f \rightarrow \min,\end{aligned}$$

where

$$t_f = \begin{cases} t, & \text{if } \sqrt{\sum_{\alpha} (x_{\alpha}(t) - x_{\alpha}^j)^2} < \varepsilon, \quad \alpha = 1, 3, 5. \\ t^+, & \text{otherwise} \end{cases}$$

The parameters of model $m_0 = 1$, $I_1/l = 0.03$, $I_3/l = 0.03$, $g_0 = 9.81$, $u^- = 1.5$, $u^+ = 4$, $x_7^- = -0.1$, $x_7^+ = 0.1$, $x_9^- = -0.1$, $x_9^+ = 0.1$, $t^+ = 24$.

The control stabilizes the quadrotor near the points of trajectory and intelligent control switches from point to point.

For the solution of our problem we used a genetic algorithm and the methods of arithmetic and logical network operators.

For trajectory of eight points

$$\begin{aligned}P &= ((5,18,0), (10,16,0), (5,16,5), (0,16,10), \\ &(5,14,10), (10,12,10), (5,10,5), (0,10,0))\end{aligned}$$

we have received the control

$$\begin{aligned}T &= q_{10}x_4 + q_{11}(g_0 + \arctan(x_4) + \sin(x_4)) - \\ &q_9(x_3 - x_3^j) + q_{13}x_9^2 - q_{12} \operatorname{sgn}(x_3 - x_3^j) \sqrt{|x_3 - x_3^j|}, \\ u_2 - u_4 &= -q_5x_{10} - q_6x_9 + q_7x_2 + q_8(x_1 - x_1^j) + q_{14}, \\ u_1 - u_3 &= -q_1x_8 - q_2x_7 + q_3x_6 + q_4(x_5 - x_5^j),\end{aligned}$$

where $q_1 = 0.134$, $q_2 = 0.375$, $q_3 = 0.066$, $q_4 = 0.041$, $q_5 = 0.118$, $q_6 = 0.328$, $q_7 = 0.058$, $q_8 = 0.035$, $q_9 = 13.81$, $q_{10} = 13.53$, $q_{11} = 0.8906$, $q_{12} = 3.51$, $q_{13} = 0.247$, $q_{14} = 0.001$.

We have received the intelligent control

$$v = y_1 y_3 y_5 z_1 \vee z_{\varepsilon},$$

where $y_{\alpha} = \theta(d_{\alpha} - |x_{\alpha} - x_{\alpha}^j|)$, $z_1 = \theta((x_1^{j+1} - x_1)x_2)$,

$$z_{\varepsilon} = \theta\left(\varepsilon - \sqrt{\sum_{\alpha} (x_{\alpha} - x_{\alpha}^j)^2}\right), \quad \theta(A) = \begin{cases} 1, & \text{if } A \geq 0 \\ 0, & \text{otherwise} \end{cases},$$

$$d_{\alpha} = 0.5, \quad \alpha = 1, 3, 5, \quad \varepsilon = 0.2.$$

The set of Pareto optimal solutions is presented in Fig 3.

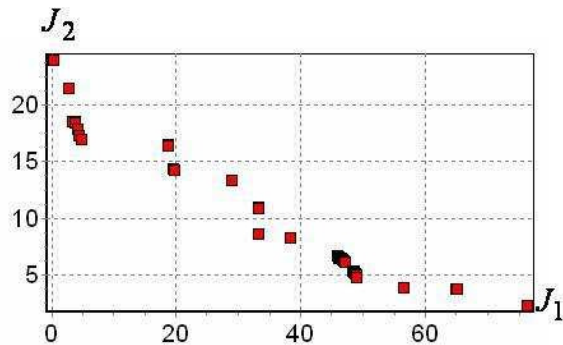


Figure 3. A Pareto set

The results of simulation for one of solutions from the Pareto set are presented in Fig. 4-6. Red points indicate the set space trajectory. For received controls we have the following values of quality criteria $J_1 = 3.38$, $J_2 = 18.61$.

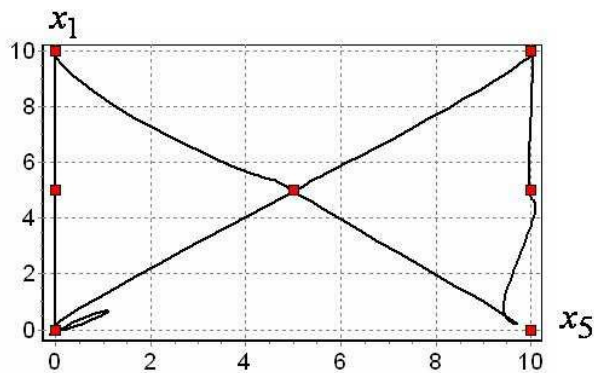


Figure 4. A projection of the trajectory on a plane $\{x_1, x_5\}$

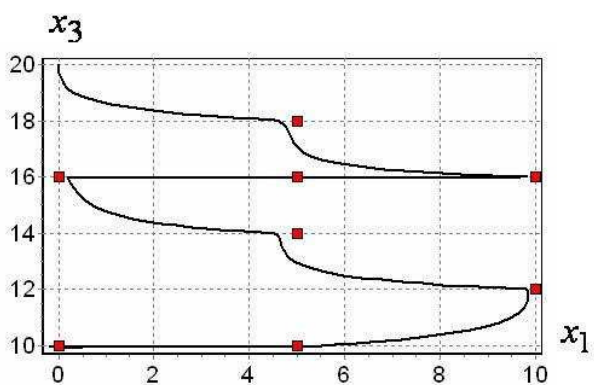


Figure 5. A projection of the trajectory on a plane $\{x_3, x_1\}$

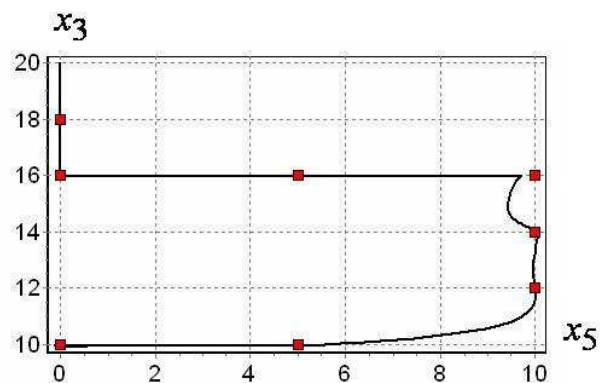


Figure 6. A projection of the trajectory on a plane $\{x_3, x_5\}$

VIII. CONCLUSION

We obtained an intelligent control by the numerical network operator method. The control provides the movement of unmanned helicopter over the trajectory points, and the time of movement is less than 20 seconds.

IX. ACKNOWLEDGMENTS

This work is supported by Russian Found of Basic Research, project No 11-08-00532-a.

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